STATIS TAK TENTU
SLOPE-DEFLECTION METHOD (FRAME)

ANALISIS STRUKTUR – TSI204 (3 sks)

Pertemuan 13

Statically indeterminate structure

Force Method

- Developed by James Clerk Maxwell (1864) and refined by Otto Mohr & Heinrich Müller-Breslau
- Limited ony 2-3 redundant reaction
- Need principle of superposition, compatibility equation and equilibrium equation

Displacement Method

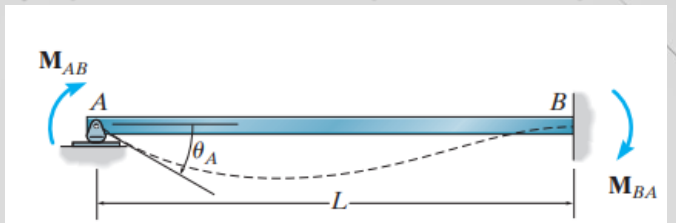
- Slope Deflection Equation (Clayperon)
- Distribution Moment
- Stiffness Matrix Method

Persamaan Slope-Deflection

- menghubungkan antara sudut rotasi (slope) dan lendutan (deflection) dengan beban yang bekerja

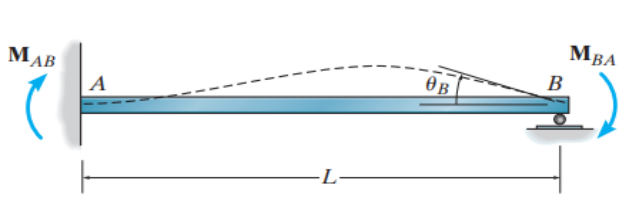
Slope

$$M_{AB} = \frac{4EI}{L}\theta_A$$



$$M_{BA} = \frac{2EI}{L}\theta_A$$

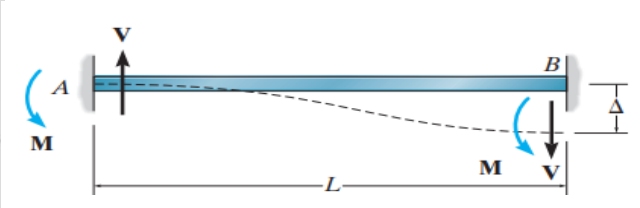
$$M_{AB} = \frac{2EI}{L}\theta_B$$



$$M_{BA} = \frac{4EI}{L}\theta_B$$

Deflection

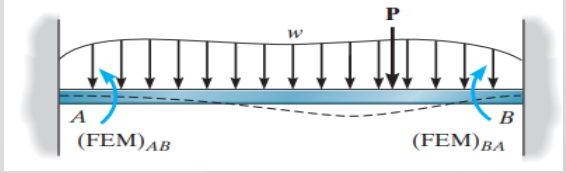
$$M_{AB} = \frac{-6EI}{L^2}\Delta$$



$$M_{BA} = \frac{-6EI}{L^2}\Delta$$

Load

$$M_{AB} = (FEM)_{AB}$$



$$M_{BA} = (FEM)_{BA}$$

+

$$M_{AB} = 2E\left(\frac{I}{L}\right)\left[2\theta_A + \theta_B - 3\left(\frac{\Delta}{L}\right)\right] + (\text{FEM})_{AB}$$

$$M_{BA} = 2E\left(\frac{I}{L}\right)\left[2\theta_B + \theta_A - 3\left(\frac{\Delta}{L}\right)\right] + (\text{FEM})_{BA}$$

Persamaan Slope-Deflection

For Internal span, or end span with far end fixed

$$M_N = 2Ek\left(2\theta_N + \theta_F - 3\Psi\right) + (\mathrm{FEM})_N$$

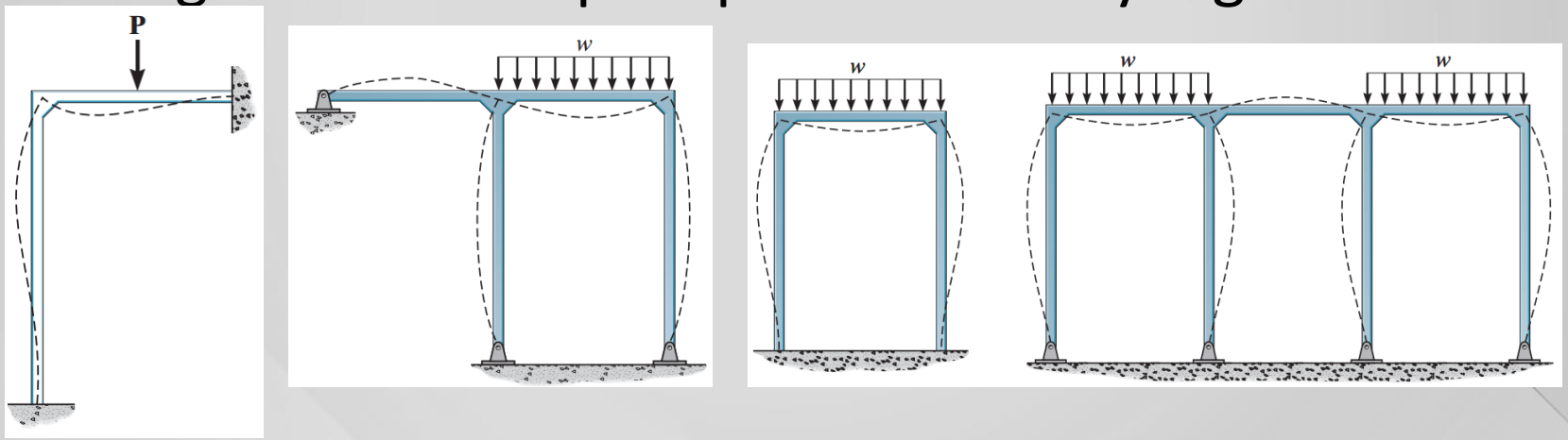
Only for end span, with far end pinned or roller support

$$M_N = 3Ek\left(\theta_N - \Psi\right) + (\mathrm{FEM})_N$$

- Momen dan sudut rotasi bernilai positif apabila memiliki arah putar searah jarum jam.
- Lendutan Δ dianggap bernilai positif apabila mengakibatkan balok berputar sebesar sudut ψ searah jarum jam.

Analisis Portal Tak Bergoyang Metode Slope-Deflection

- Suatu portal dikategorikan sebagai portal tak bergoyang apabila :
  1. Disediakan tumpuan yang cukup untuk menahan goyangan
  2. Memiliki geometri dan pola pembebanan yang simetris

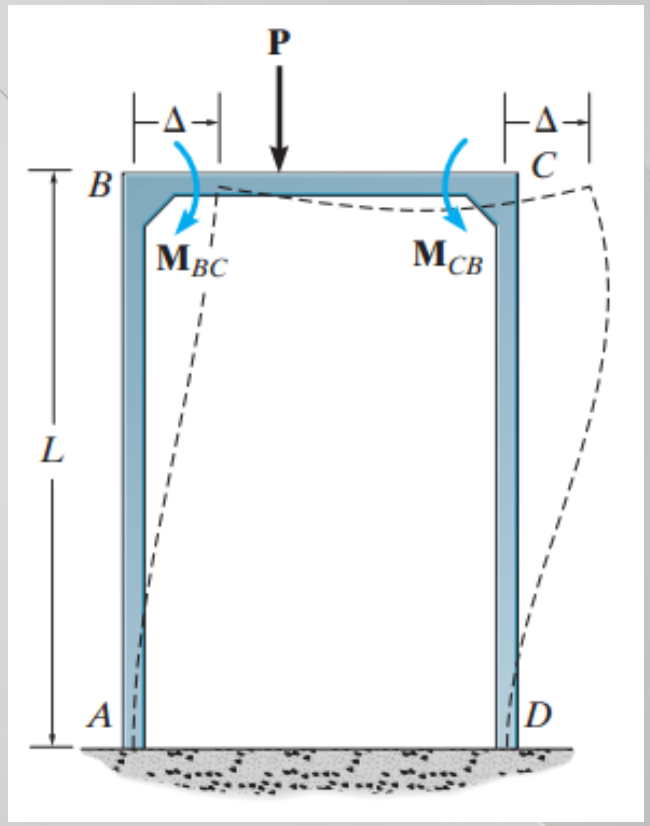


Contoh-contoh Portal Tak Bergoyang (No Sidesway)

Analisis Portal Bergoyang Metode Slope-Deflection

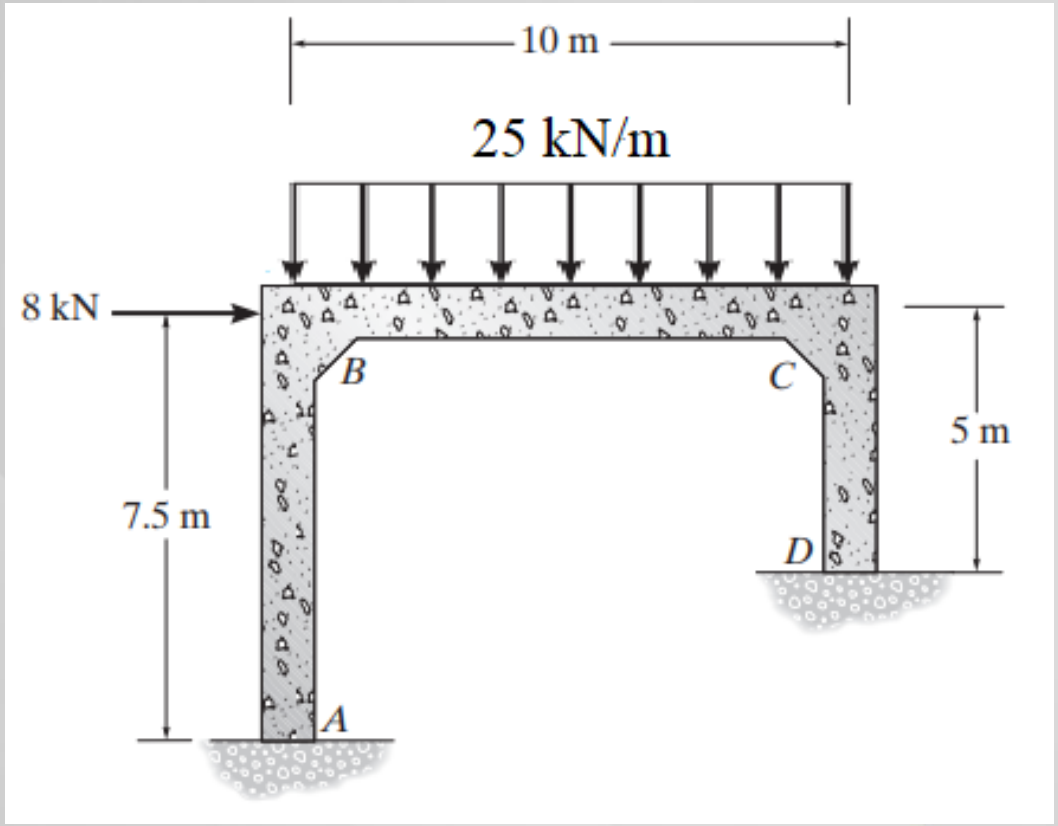
Suatu struktur portal akan bergoyang ke samping apabila

- geometri atau pembebanan yang terjadi tidak simetri
- Tidak tersedia tumpuan cukup untuk menahan goyangan
- Ada nya beban lateral (akibat beban gempa dan/atau beban angin)

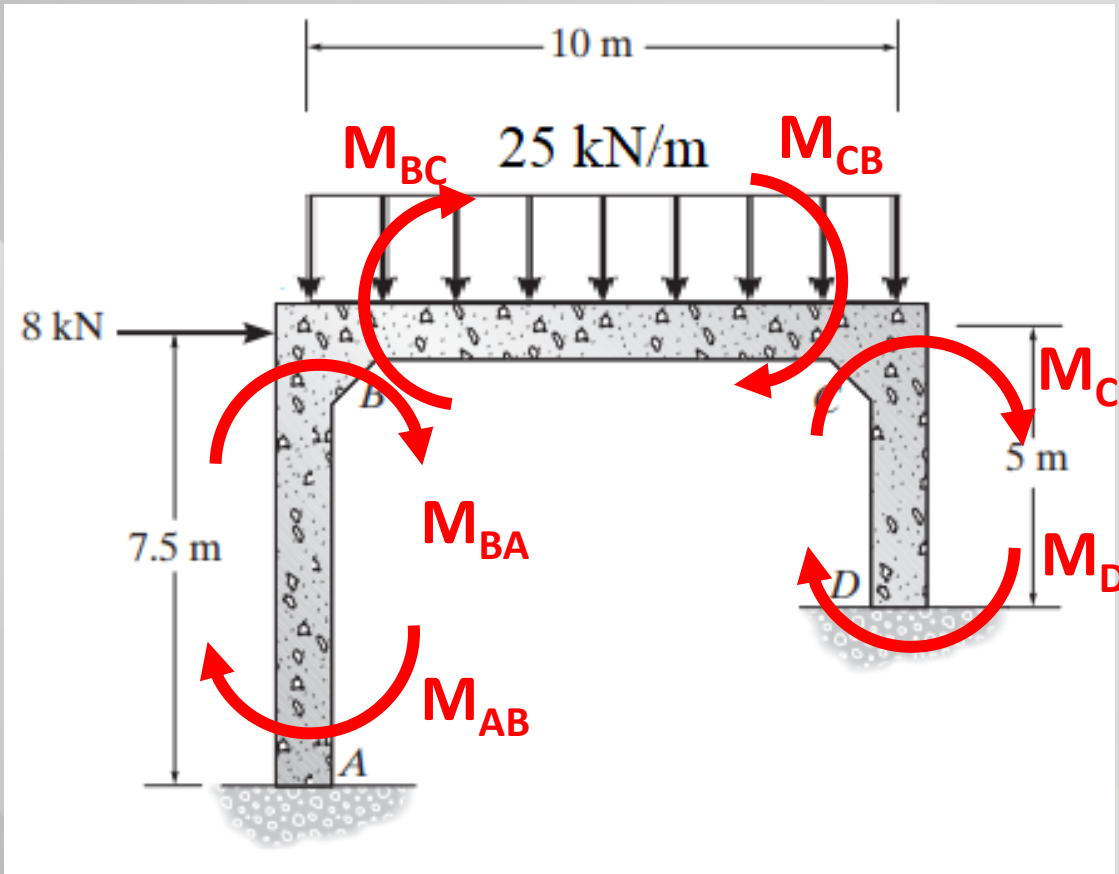


Simpangan dalam arah lateral (Δ) diasumsikan sama besar pada lantai yang sama

Determine the reactions at the supports. Assume A and D are fixed. EI is constant.



Determine the reactions at the supports. Assume A fixed and D are pins. EI is constant.



M_{AB} =

M_{BA} =

M_{BC} =

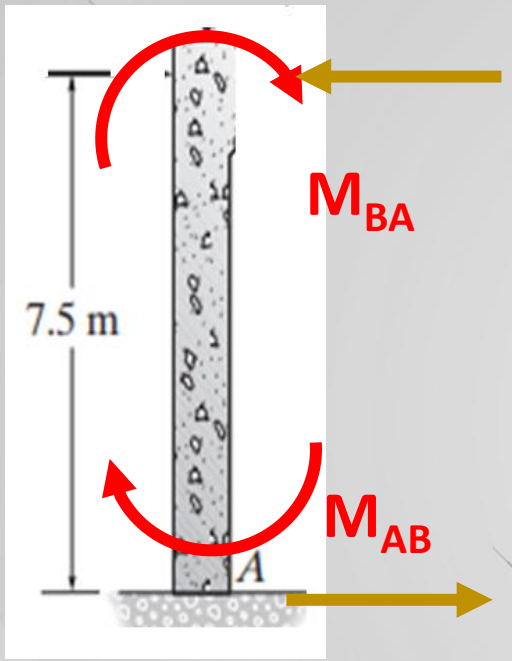
M_{CB} =

M_{CD} =

M_{DC} =

$$M_{BA} + M_{BC} = 0$$

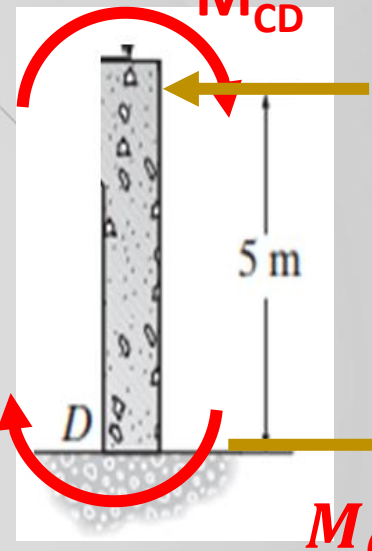
$$M_{CB} + M_{CD} = 0$$



M_{BA}

M_{AB}

$$\frac{M_{AB} + M_{BA}}{4}$$



M_{CD}

M_{DC}

$$\frac{M_{CD} + MDC}{4}$$

$\Sigma H = 0$

$\theta_B =$

$\theta_C =$

$\Delta =$

$M_{AB} =$

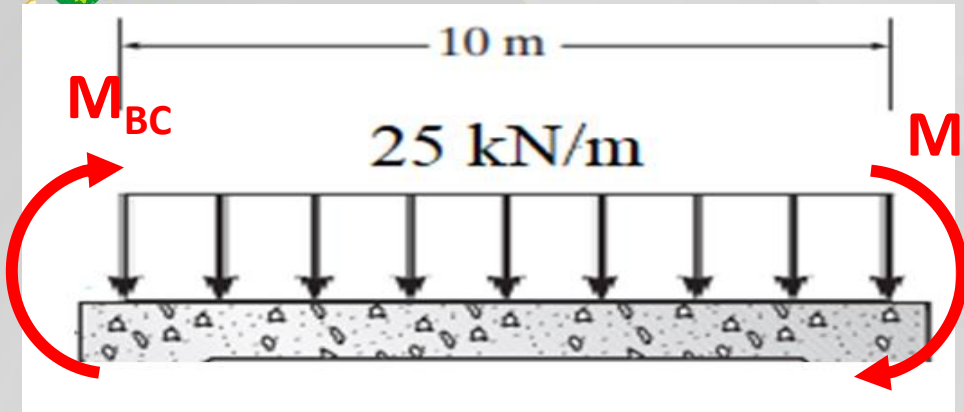
$M_{BA} =$

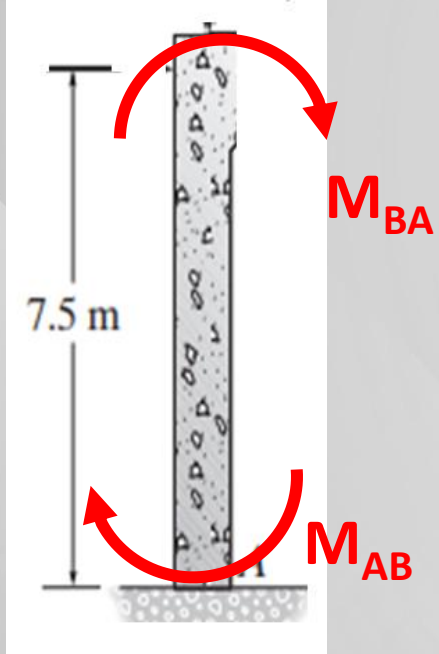
$M_{BC} =$

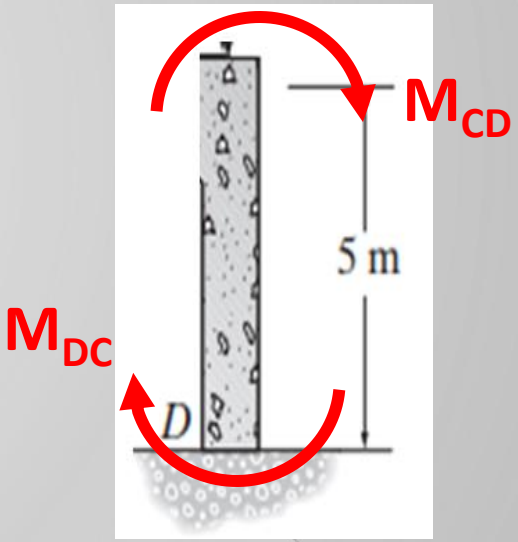
$M_{CB} =$

$M_{CD} =$

$M_{DC} =$







Analisis Portal Tak Bergoyang Metode Slope-Deflection

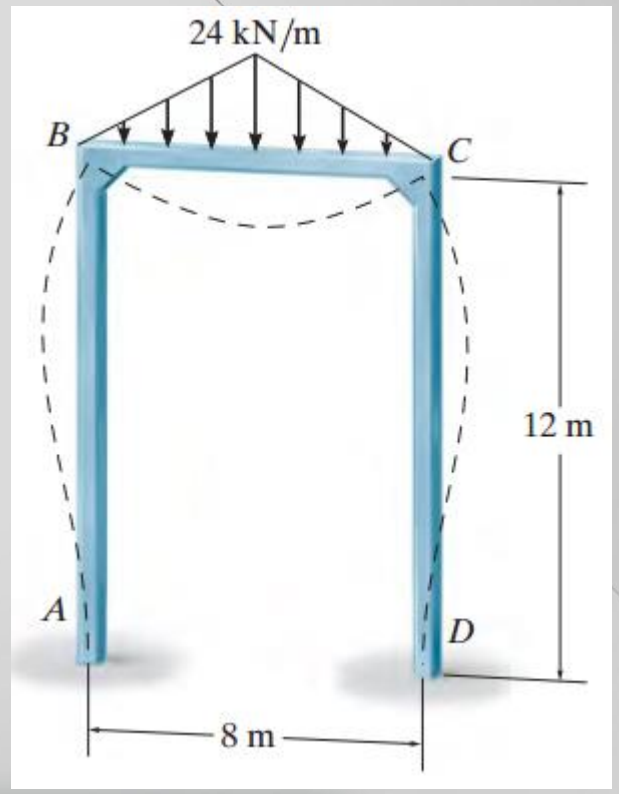
Tentukan momen pada tiap titik kumpul, apabila EI konstan

$$(\text{FEM})_{BC} = -\frac{5wL^2}{96} = -\frac{5(24)(8)^2}{96} = -80\text{kN}\cdot\text{m}$$

$$(\text{FEM})_{CB} = \frac{5wL^2}{96} = \frac{5(24)(8)^2}{96} = 80\text{kN}\cdot\text{m}$$

$$\theta_A = \theta_D = 0$$

$$\psi_{AB} = \psi_{BC} = \psi_{CD} = 0 \quad \text{(tak bergoyang)}$$



Analisis Portal Tak Bergoyang Metode Slope-Deflection

$$M_N = 2Ek(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AB} = 2E\left(\frac{I}{12}\right)[2(0) + \theta_B - 3(0)] + 0 = 0{,}1667EI\theta_B$$

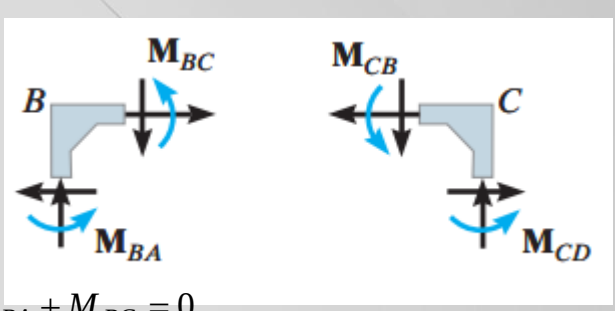
$$M_{BA} = 2E\left(\frac{I}{12}\right)[2\theta_B + 0 - 3(0)] + 0 = 0{,}333EI\theta_B$$

$$M_{BC} = 2E\left(\frac{I}{8}\right)[2\theta_B + \theta_C - 3(0)] - 80 = 0{,}5EI\theta_B + 0{,}25EI\theta_C - 80$$

$$M_{CB} = 2E\left(\frac{I}{8}\right)[2\theta_C + \theta_B - 3(0)] + 80 = 0{,}5EI\theta_C + 0{,}25EI\theta_B + 80$$

$$M_{CD} = 2E\left(\frac{I}{12}\right)[2\theta_C + 0 - 3(0)] + 0 = 0{,}333EI\theta_C$$

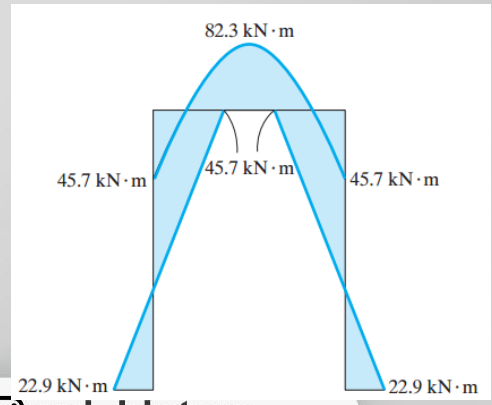
$$M_{DC} = 2E\left(\frac{I}{12}\right)[2(0) + \theta_C - 3(0)] + 0 = 0{,}1667EI\theta_C$$



$$M_{BA} + M_{BC} = 0$$

$$M_{CB} + M_{CD} = 0$$

$$\left.\begin{aligned} 0{,}833EI\theta_B + 0{,}25EI\theta_C &= 80 \\ 0{,}25EI\theta_B + 0{,}833EI\theta_C &= -80 \end{aligned}\right\} \theta_B = -\theta_C = \frac{137{,}1}{EI}$$



Analisis Portal Tak Bergoyang Metode Slope-Deflection

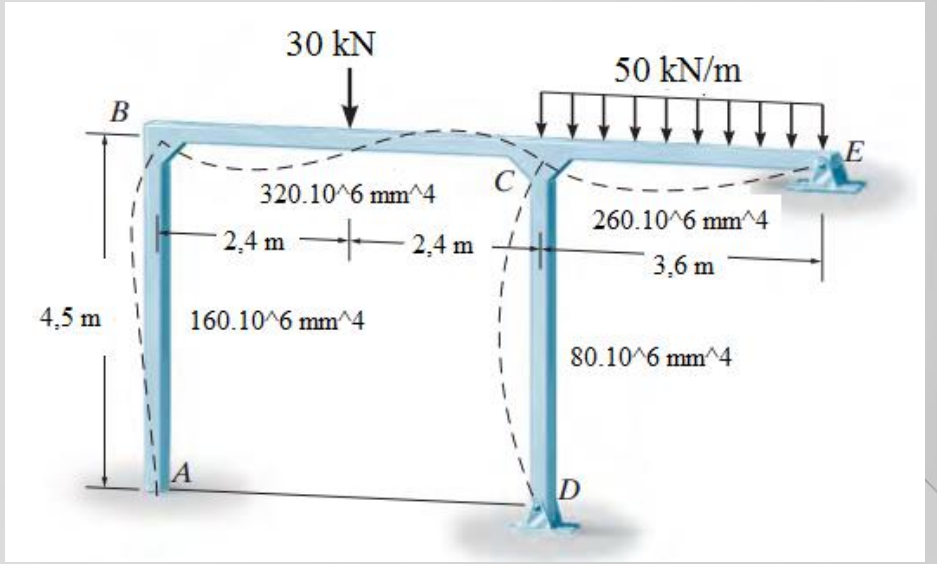
Tentukan momen pada tiap titik kumpul, gunakan E = 200 GPa

$$k_{AB} = \frac{160(10^6)(10^{-12})}{4{,}5} = 35{,}56(10^{-6})\text{m}^3$$

$$k_{BC} = \frac{320(10^6)(10^{-12})}{4{,}8} = 66{,}67(10^{-6})\text{m}^3$$

$$k_{CD} = \frac{80(10^6)(10^{-12})}{4{,}5} = 17{,}78(10^{-6})\text{m}^3$$

$$k_{CE} = \frac{260(10^6)(10^{-12})}{3{,}6} = 72{,}23(10^{-6})\text{m}^3$$



Analisis Portal Tak Bergoyang Metode Slope-Deflection

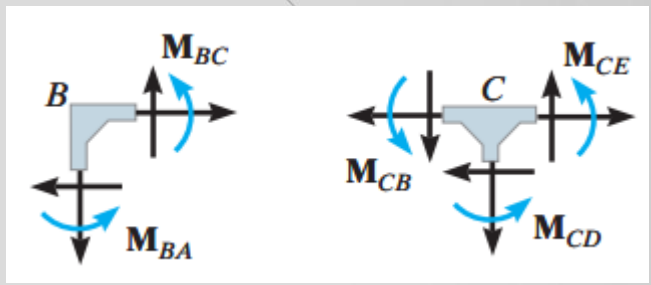
$$(\text{FEM})_{BC} = -\frac{PL}{8} = -\frac{30(4,8)}{8} = -18\,\text{kN}\cdot\text{m}$$

$$(\text{FEM})_{CB} = \frac{PL}{8} = \frac{30(4,8)}{8} = 18\,\text{kN}\cdot\text{m}$$

$$(\text{FEM})_{CE} = -\frac{wL^2}{8} = -\frac{50(3,6)^2}{8} = -81\,\text{kN}\cdot\text{m}$$

$$\theta_A = 0$$

$$\psi_{AB} = \psi_{BC} = \psi_{CD} = \psi_{CE} = 0 \quad (\text{tak bergoyang})$$



$$M_{BA} + M_{BC} = 0$$

$$M_{CB} + M_{CD} + M_{CE} = 0$$

$$\left.\begin{aligned} 61.759,3\theta_B + 20.138,9\theta_C &= 12 \\ 20.138,9\theta_B + 81.059,0\theta_C &= 42 \end{aligned}\right\} \Rightarrow \begin{aligned} \theta_B &= 2,758(10^{-5})\,\text{rad} \\ \theta_C &= 5,113(10^{-4})\,\text{rad} \end{aligned}$$

$$M_{AB} = 0,444\,\text{kN}\cdot\text{m}$$

$$M_{BA} = 0,888\,\text{kN}\cdot\text{m}$$

$$M_{BC} = -0,888\,\text{kN}\cdot\text{m}$$

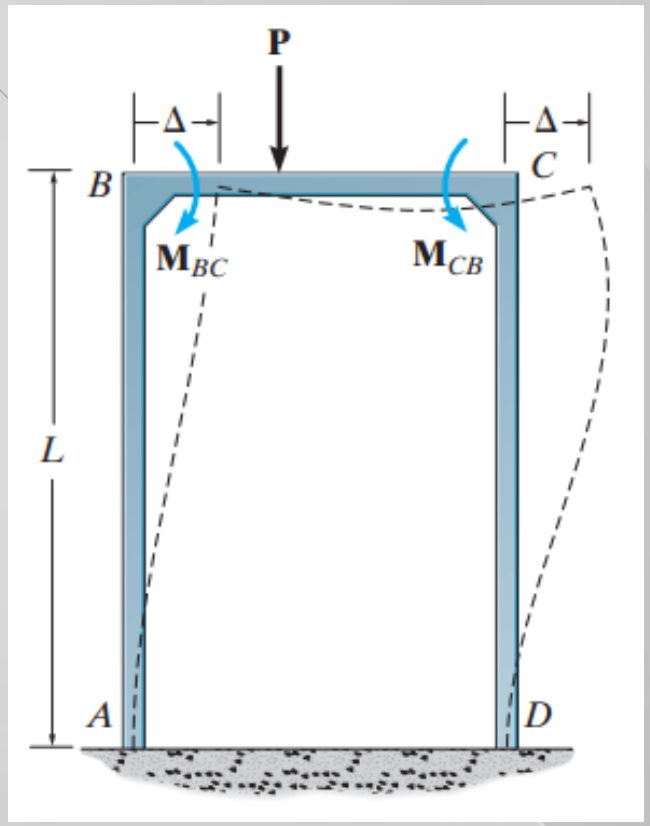
$$M_{CB} = 49,7\,\text{kN}\cdot\text{m}$$

$$M_{CD} = 6,18\,\text{kN}\cdot\text{m}$$

$$M_{CE} = -55,9\,\text{kN}\cdot\text{m}$$

Analisis Portal Bergoyang Metode Slope-Deflection

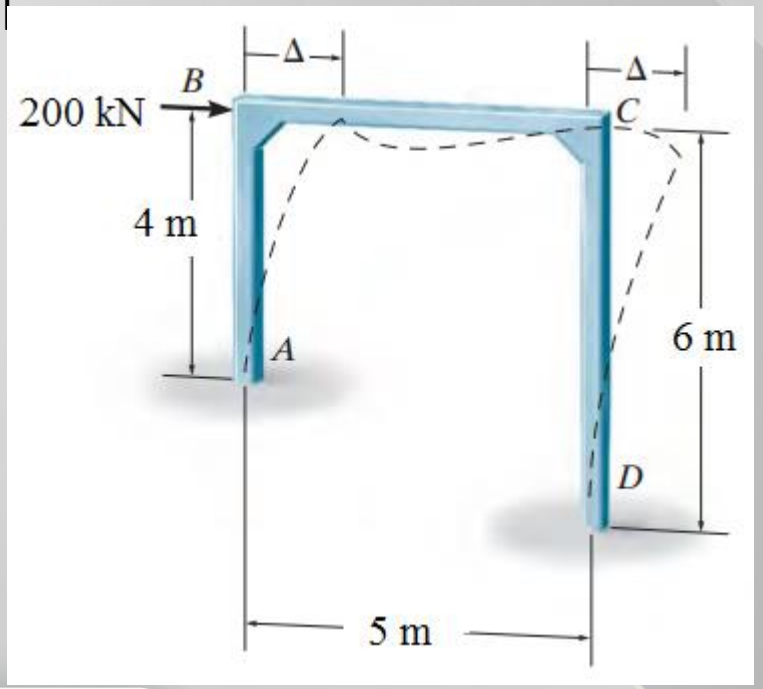
- Suatu struktur portal akan bergoyang ke samping apabila geometri atau pembebanan yang terjadi tidak simetri
- Pada portal di samping beban *P* menimbulkan momen M_{BC} dan M_{CB} pada titik kumpul B dan C
- M_{BC} cenderung memindahkan titik B ke kanan, sedangkan M_{CB} cenderung memindahkan titik C ke kiri
- Karena M_{BC} lebih besar daripada M_{CB} sebagai hasilnya portal akan timbul simpangan sebesar D ke arah kanan, pada titik B maupun C



Analisis Portal Bergoyang Metode Slope-Deflection

Tentukan momen pada tiap titik kumpul, anggaplah EI konstan.

- Struktur termasuk portal bergoyang karena baik geometri dan beban tidak simetri
- Beban bekerja pada titik B, sehingga tidak ada FEM
- Titik B dan C mengalami simpangan sama besar yaitu Δ
- Sehingga $\psi_{AB} = \Delta/4$ dan $\psi_{DC} = \Delta/6$
- Keduanya positif karena batang AB dan CD berotasi searah jarum jam
- $\psi_{AB} = (6/4)\psi_{DC}$



Analisis Portal Bergoyang Metode Slope-Deflection

$$M_{AB} = 2E\left(\frac{I}{4}\right)\left[2(0) + \theta_B - 3\left(\frac{6}{4}\psi_{DC}\right)\right] + 0 = EI\left(0{,}5\theta_B - 2{,}25\psi_{DC}\right)$$

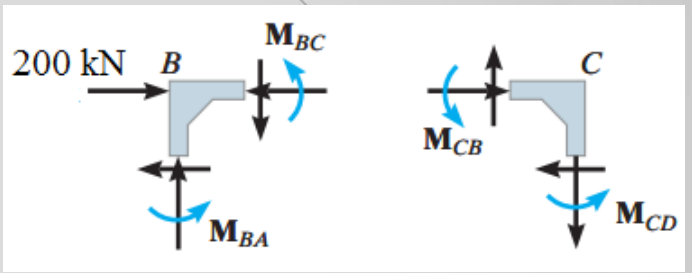
$$M_{BA} = 2E\left(\frac{I}{4}\right)\left[2\theta_B + 0 - 3\left(\frac{6}{4}\psi_{DC}\right)\right] + 0 = EI\left(1{,}0\theta_B - 2{,}25\psi_{DC}\right)$$

$$M_{BC} = 2E\left(\frac{I}{5}\right)\left[2\theta_B + \theta_C - 3(0)\right] + 0 = EI\left(0{,}8\theta_B + 0{,}4\theta_C\right)$$

$$M_{CB} = 2E\left(\frac{I}{4}\right)\left[2\theta_C + \theta_B - 3(0)\right] + 0 = EI\left(0{,}8\theta_C + 0{,}4\theta_B\right)$$

$$M_{CD} = 2E\left(\frac{I}{6}\right)\left[2\theta_C + 0 - 3(\psi_{DC})\right] + 0 = EI\left(0{,}667\theta_C - 1{,}0\psi_{DC}\right)$$

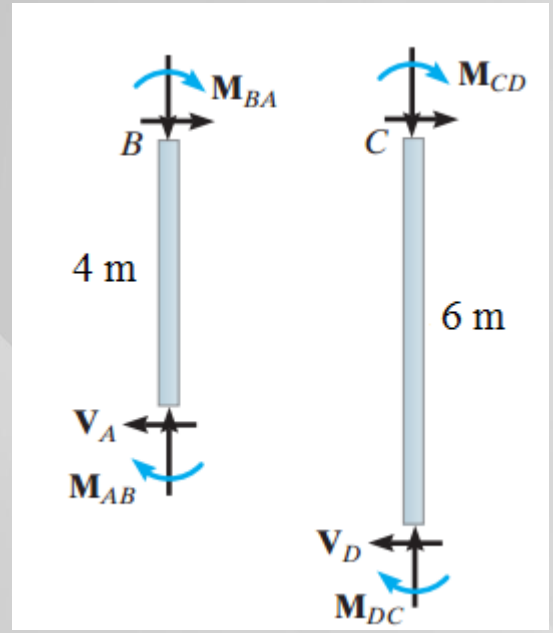
$$M_{DC} = 2E\left(\frac{I}{6}\right)\left[2(0) + \theta_C - 3(\psi_{DC})\right] + 0 = EI\left(0{,}333\theta_C - 1{,}0\psi_{DC}\right)$$



$$M_{BA} + M_{BC} = 0$$

$$M_{CB} + M_{CD} = 0$$

Analisis Portal Bergoyang Metode Slope-Deflection



$$\Sigma M_B = 0 \qquad V_A = -\frac{M_{AB} + M_{BA}}{4}$$

$$\Sigma M_C = 0 \qquad V_D = -\frac{M_{DC} + M_{CD}}{6}$$

$$\Sigma F_x = 0 \qquad 200 - V_A - V_D = 0 \quad \Rightarrow \quad 200 + \frac{M_{AB} + M_{BA}}{4} + \frac{M_{DC} + M_{CD}}{6} = 0$$

Sehingga diperoleh 3 buah persamaan :

$$\left.\begin{aligned} 1{,}8\theta_B + 0{,}4\theta_C - 2{,}25\psi_{DC} &= 0 \\ 0{,}4\theta_B + 1{,}467\theta_C - \psi_{DC} &= 0 \\ 1{,}5\theta_B + 0{,}667\theta_C - 5{,}833\psi_{DC} &= \frac{800}{EI} \end{aligned}\right\} \Rightarrow \begin{aligned} EI\theta_B &= 243{,}78 \\ EI\theta_C &= 75{,}66 \\ EI\psi_{DC} &= 208{,}48 \end{aligned}$$

$$M_{AB} = -347\ \text{kN}\cdot\text{m} \qquad M_{BA} = -225\ \text{kN}\cdot\text{m}$$

$$M_{BC} = 225\ \text{kN}\cdot\text{m} \qquad M_{CB} = 158\ \text{kN}\cdot\text{m}$$

$$M_{CD} = -158\ \text{kN}\cdot\text{m} \qquad M_{DC} = -183\ \text{kN}\cdot\text{m}$$

Analisis Portal Bergoyang Metode Slope-Deflection

Tentukan momen pada tiap titik kumpul, anggaplah EI konstan. A dan D jepit, serta C adalah berupa sendi

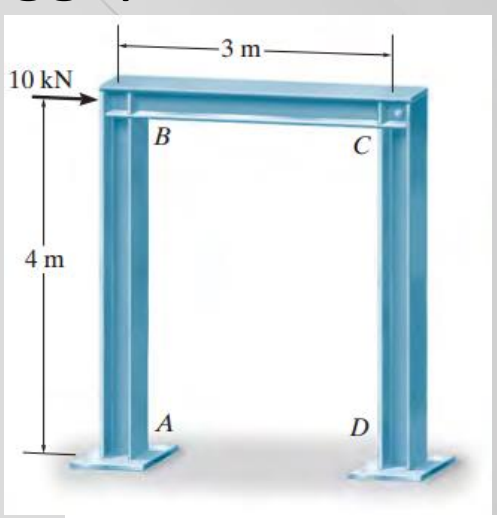
- $\psi = \psi_{AB} = \psi_{DC} = \Delta/4$
- $\theta_A = \theta_D = 0$

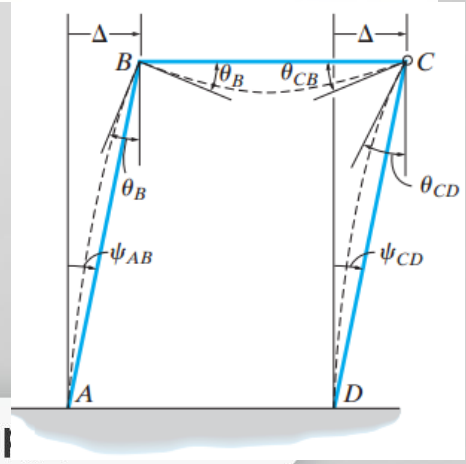
$$M_{AB} = 2E\left(\frac{I}{4}\right)\left[2(0) + \theta_B - 3\psi\right] + 0 = 0{,}5EI\theta_B - 1{,}5EI\psi$$

$$M_{BA} = 2E\left(\frac{I}{4}\right)\left[2\theta_B + 0 - 3\psi\right] + 0 = EI\theta_B - 1{,}5EI\psi$$

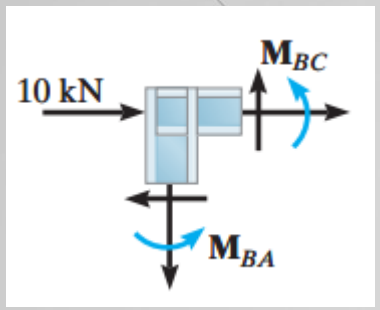
$$M_{BC} = 3E\left(\frac{I}{3}\right)\left[\theta_B - 0\right] + 0 = EI\theta_B$$

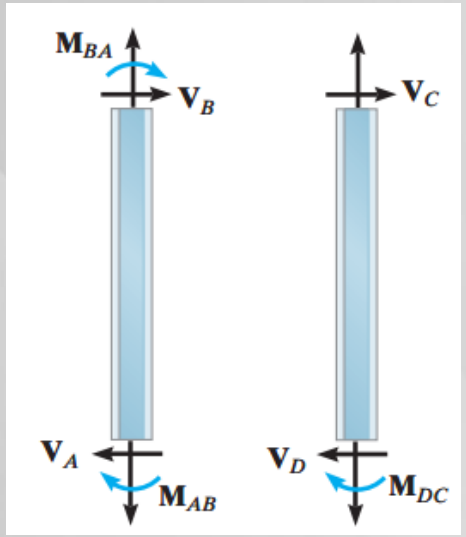
$$M_{DC} = 3E\left(\frac{I}{4}\right)\left[0 - \psi\right] + 0 = -0{,}75EI\psi$$





Analisis Portal Bergoyang Metode Slope-Deflection





$$M_{BA} + M_{BC} = 0$$

$$\Sigma M_B = 0 \qquad V_A = -\frac{M_{AB} + M_{BA}}{4}$$

$$\Sigma M_C = 0 \qquad V_D = -\frac{M_{DC}}{4}$$

$$\Sigma F_x = 0 \qquad 10 - V_A - V_D = 0 \quad \Rightarrow \quad 10 + \frac{M_{AB} + M_{BA}}{4} + \frac{M_{DC}}{4} = 0$$

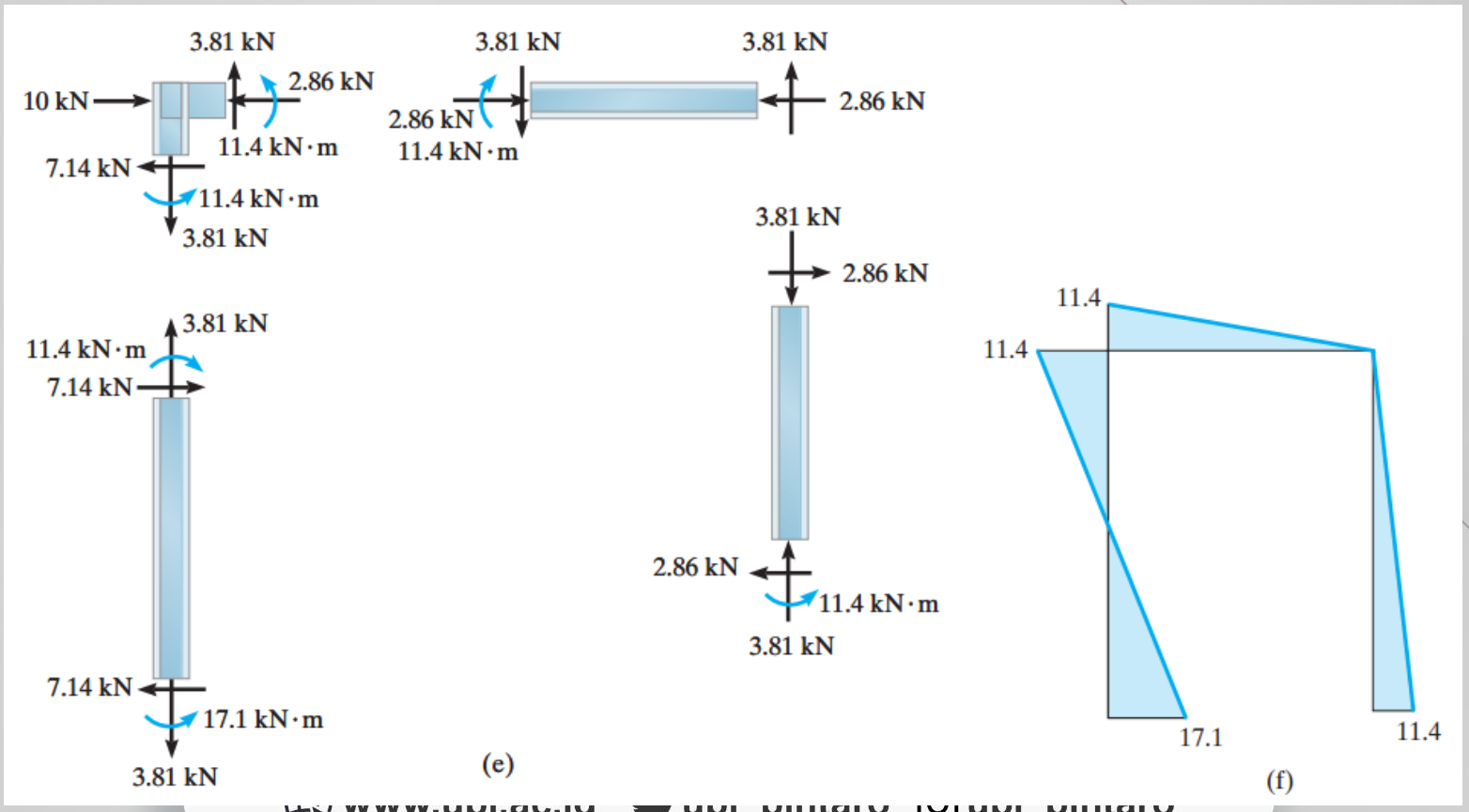
Substitusikan nilai M_{AB}, M_{BA}, M_{BC} dan M_{DC} sehingga dip

$$\theta_B = \frac{240}{21EI} \qquad \psi = \frac{320}{21\text{EI}}$$

$$M_{AB} = -17{,}1 \text{ kN}\cdot\text{m} \qquad M_{BA} = -11{,}4 \text{ kN}\cdot\text{m}$$

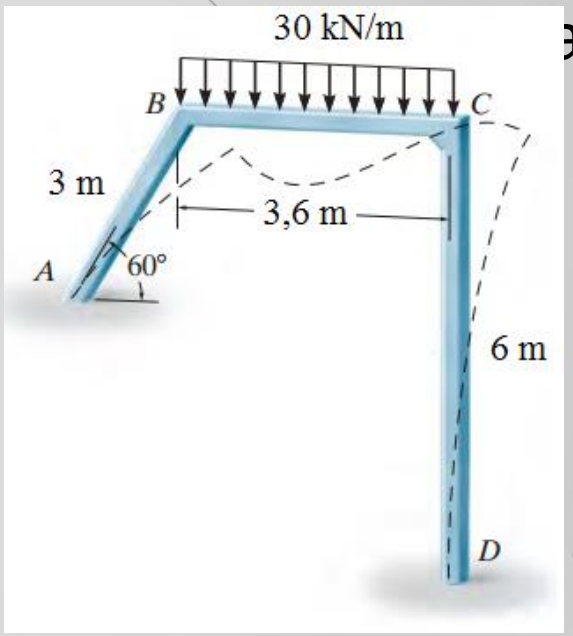
$$M_{BC} = 11{,}4 \text{ kN}\cdot\text{m} \qquad M_{DC} = -11{,}4 \text{ kN}\cdot\text{m}$$

Analisis Portal Bergoyang Metode Slope-Deflection



Example 11.10

Tentukan momen pada tiap titik kumpul, a an.



$$(\text{FEM})_{BC} = -\frac{wL^2}{12} = -\frac{30(3,6)^2}{12} = -32,4\text{kN}\cdot\text{m}$$

$$(\text{FEM})_{CB} = \frac{wL^2}{12} = \frac{30(3,6)^2}{12} = 32,4\text{kN}\cdot\text{m}$$

$$\psi_1 = \frac{\Delta_1}{3} \qquad \psi_2 = -\frac{\Delta_2}{3,6} \qquad \psi_3 = \frac{\Delta_3}{6}$$

$$\Delta_2 = 0,5\Delta_1 \qquad \Delta_3 = 0,866\Delta_1 \qquad \Rightarrow \psi_2 = -0,417\psi_1 \qquad \psi_3 = 0,433\psi_1$$

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$$M_{AB} = 2E\left(\frac{I}{3}\right)\left[2(0) + \theta_B - 3\psi_1\right] + 0$$

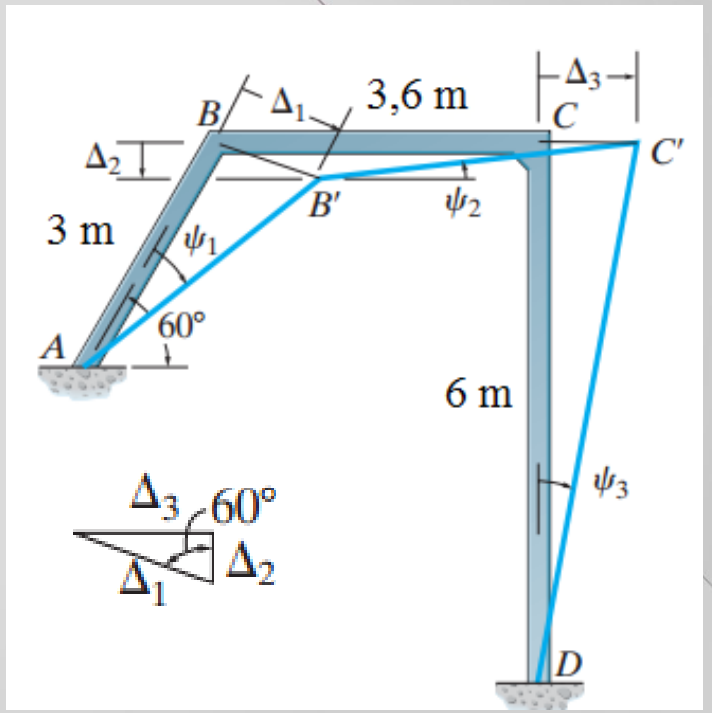
$$M_{BA} = 2E\left(\frac{I}{3}\right)\left[2\theta_B + 0 - 3\psi_1\right] + 0$$

$$M_{BC} = 2E\left(\frac{I}{3{,}6}\right)\left[2\theta_B + \theta_C - 3(-0{,}417\psi_1)\right] - 32{,}4$$

$$M_{CB} = 2E\left(\frac{I}{3{,}6}\right)\left[2\theta_C + \theta_B - 3(-0{,}417\psi_1)\right] + 32{,}4$$

$$M_{CD} = 2E\left(\frac{I}{6}\right)\left[2\theta_C + 0 - 3(0{,}433\psi_1)\right] + 0$$

$$M_{DC} = 2E\left(\frac{I}{6}\right)\left[2(0) + \theta_C - 3(0{,}433\psi_1)\right] + 0$$



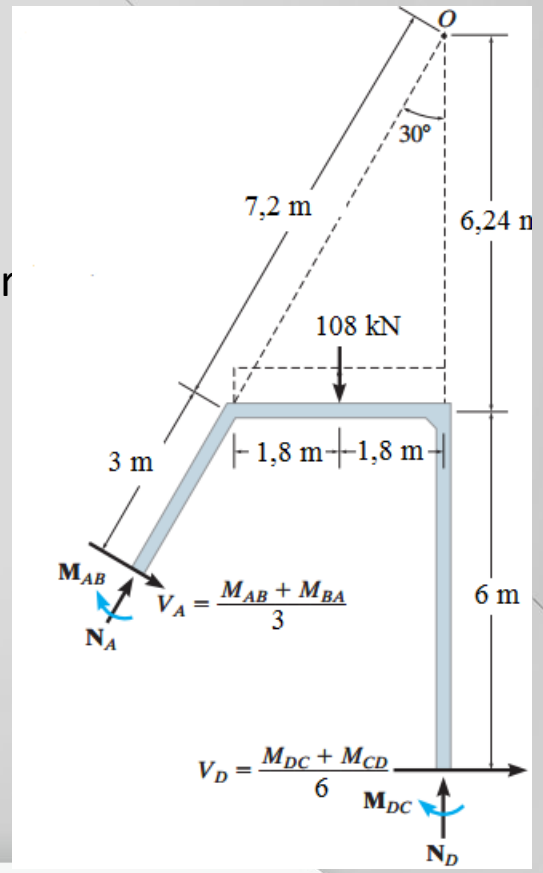
Analisis Portal Bergoyang Metode Slope-Deflection

Dari kesetimbangan momen di titik B dan C :

$$M_{BA} + M_{BC} = 0$$

$$M_{CD} + M_{CB} = 0$$

Persamaan ketiga diperoleh dengan mengambil jumlahan momen ter



$$\Sigma M_O = 0$$

$$M_{AB} + M_{DC} - \left(\frac{M_{AB} + M_{BA}}{3}\right)(10,2) - \left(\frac{M_{DC} + M_{CD}}{6}\right)(12,24) - 108(1,8) = 0$$

$$-2,4M_{AB} - 3,4M_{BA} - 2,04M_{CD} - 1,04M_{DC} - 194,4 = 0$$

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Diperoleh 3 buah persamaan :

$$\left.\begin{aligned} 0{,}733\theta_B + 0{,}167\theta_C - 0{,}392\psi_1 &= 9{,}72/EI \\ 0{,}167\theta_B + 0{,}533\theta_C + 0{,}0784\psi_1 &= -9{,}72/EI \\ -1{,}840\theta_B - 0{,}512\theta_C + 3{,}880\psi_1 &= 58{,}32/EI \end{aligned}\right\} \Rightarrow \begin{aligned} EI\theta_B &= 35{,}51 \\ EI\theta_C &= -33{,}33 \\ EI\psi_1 &= 27{,}47 \end{aligned}$$

substitusi kembali ke persamaan awal sehingga diperoleh :

$M_{AB} = -31{,}3\ \text{kN}\cdot\text{m}$ $M_{BC} = 7{,}60\ \text{kN}\cdot\text{m}$ $M_{CD} = -34{,}2\ \text{kN}\cdot\text{m}$

$M_{BA} = -7{,}60\ \text{kN}\cdot\text{m}$ $M_{CB} = 34{,}2\ \text{kN}\cdot\text{m}$ $M_{DC} = -23\ \text{kN}\cdot\text{m}$