Universitas Pembangunan Jaya

Integrity, Professionalism, & Entrepreneurship

Mata Kuliah : Analisis Struktur
Kode : CIV - 209
SKS : 4 SKS

Deformasi Elastis Struktur Balok dan Portal

(Conjugate Beam Method)

Pertemuan - 5

- Kemampuan Akhir yang Diharapkan
  - Mahasiswa dapat menganalisis deformasi struktur balok dan Portal dengan metode Conjugate Beam
- Sub Pokok Bahasan :
  - Metode *Conjugate beam*

- Metode balok konjugasi dikembangkan oleh H. Műller-Breslau di tahun 1865.
- Metode ini hampir sama dengan metode luas momen yang telah dibahas sebelumnya.
- Namun jika metode luas momen memerlukan bantuan secara grafis, maka metode balok konjugasi ini didasarkan pada prinsip-prinsip statika, sehingga jauh lebih mudah dipahami.
- Balok konjugasi merupakan balok fiktif yang memiliki panjang sama dengan balok nyatanya, yang diberi beban berupa diagram *M/EI* yang diperoleh dari hasil analisis balok nyata.

Teori I: **Sudut rotasi dari titik tertentu pada balok sebenarnya sama dengan kurva diagram Lintang (shear force) di titik yang sama pada balok konjugasi**

Teori II : **Perpindahan vertikal dari titik tertentu pada balok sebenarnya sama dengan kurva diagram Momen di titik yang sama pada balok konjugasi**

Persamaan Statika dan Mekanika Bahan

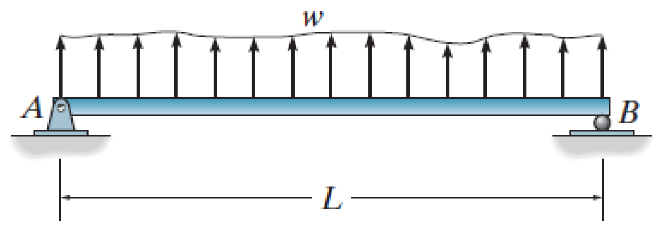
$$\frac{dV}{dx} = w \qquad \frac{d\theta}{dx} = \frac{M}{EI}$$

$$\frac{d^2M}{dx^2} = w \qquad \frac{d^2v}{dx^2} = \frac{M}{EI}$$

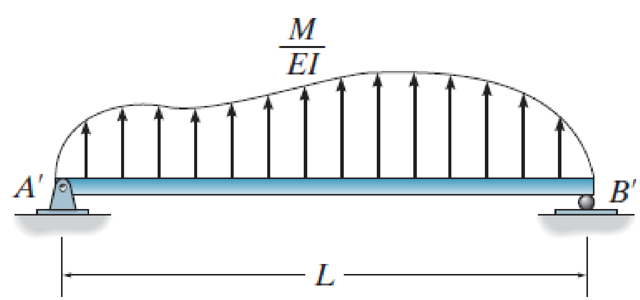
Persamaan deformasi elastis balok

$$V = \int w\, dx \quad \updownarrow \quad \theta = \int \left(\frac{M}{EI}\right) dx$$

$$M = \int \left[\int w\, dx\right] dx \quad \updownarrow \quad v = \int \left[\int \left(\frac{M}{EI}\right) dx\right] dx$$



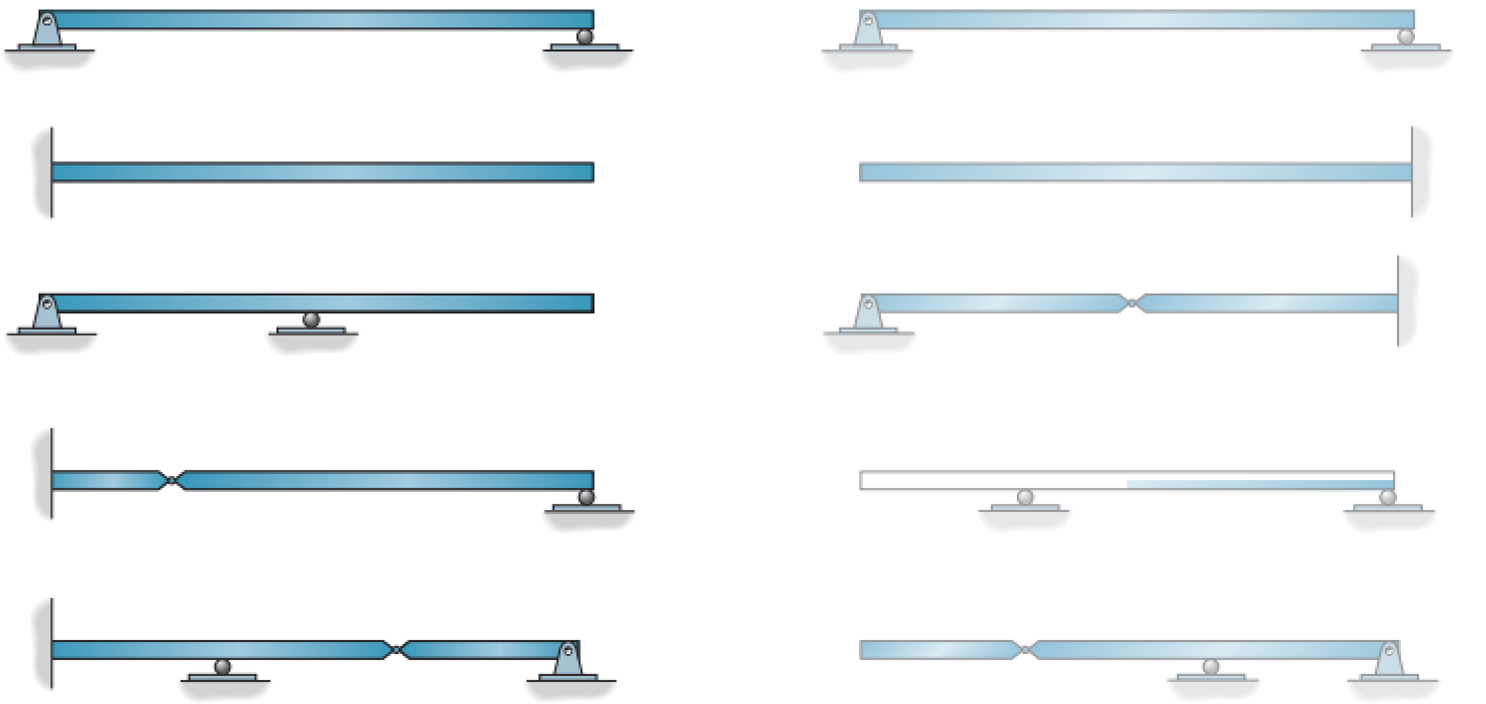
real beam



conjugate beam

- Dalam melakukan transformasi balok nyata menjadi balok konjugasi, maka harus dilakukan **penyesuaian kondisi tumpuan**

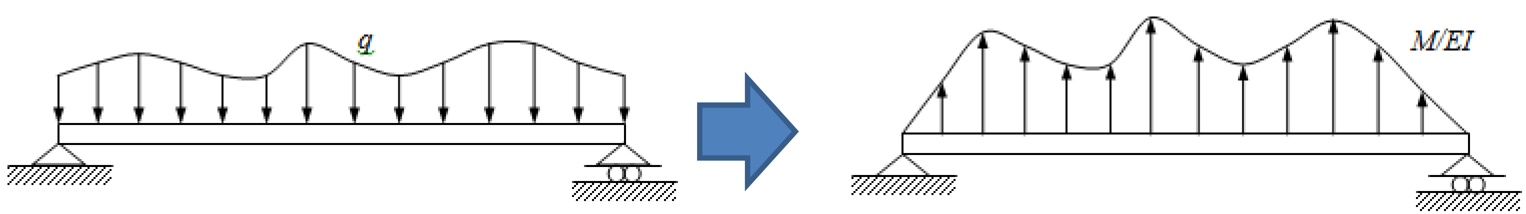
	Real Beam		Conjugate Beam	
1)	θ $\Delta = 0$	pin	V $M = 0$	pin
2)	θ $\Delta = 0$	roller	V $M = 0$	roller
3)	$\theta = 0$ $\Delta = 0$	fixed	$V = 0$ $M = 0$	free
4)	θ Δ	free	V M	fixed
5)	θ $\Delta = 0$	internal pin	V $M = 0$	hinge
6)	θ $\Delta = 0$	internal roller	V $M = 0$	hinge
7)	θ Δ	hinge	V M	internal roller



real beam

conjugate beam

- Gambar a menunjukkan suatu balok sederhana di atas dua tumpuan dengan beban merata *q*, selanjutnya balok tersebut harus ditransformasi menjadi balok konjugasi-nya seperti dalam Gambar b dengan beban merata berupa diagram *M/EI* yang dihasilkan dari pembebanan pada balok nyata.
- Apabila diagram *M/EI* dari balok nyata bernilai positif maka arah beban dari diagram *M/EI* adalah kearah atas, demikian pula sebaliknya.



(a) Balok sederhana dengan beban merata *q*

(b) Balok konjugasi dengan beban *M/EI*

Procedure for Analysis

Conjugate Beam

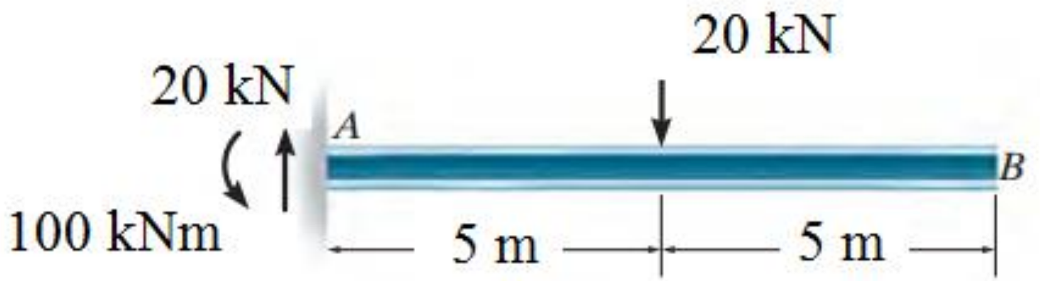
- Draw the conjugate beam for the real beam. This beam has the same length as the real beam and has corresponding supports as listed in Table 8–2.
- In general, if the real support allows a *slope*, the conjugate support must develop a *shear*; and if the real support allows a *displacement*, the conjugate support must develop a *moment*.
- The conjugate beam is loaded with the real beam's M/EI diagram. This loading is assumed to be *distributed* over the conjugate beam and is directed *upward* when M/EI is *positive* and *downward* when M/EI is *negative*. In other words, the loading always acts *away* from the beam.

Equilibrium

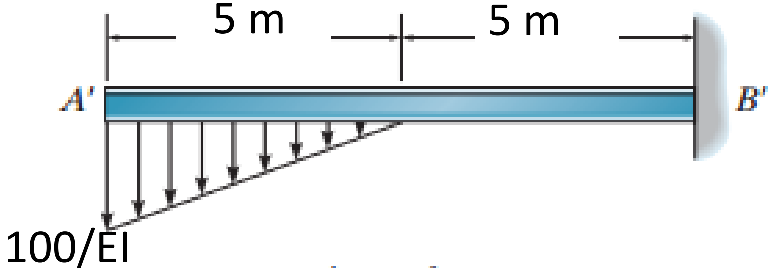
- Using the equations of equilibrium, determine the reactions at the conjugate beam's supports.
- Section the conjugate beam at the point where the slope θ and displacement Δ of the real beam are to be determined. At the section show the unknown shear V' and moment M' acting in their positive sense.
- Determine the shear and moment using the equations of equilibrium. V' and M' equal θ and Δ, respectively, for the real beam. In particular, if these values are *positive*, the *slope* is *counterclockwise* and the *displacement* is *upward*.

Example 1

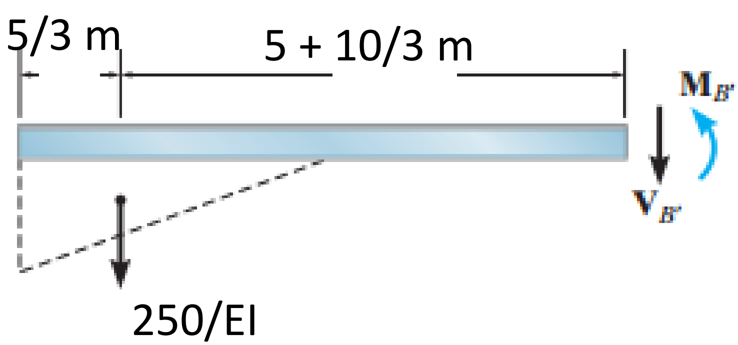
- Determine the slope and deflection at point B of the steel beam in figure.
- Use $E = 200$ GPa, $I = 475(10^6)$ mm^4



real beam
(a)



conjugate beam
(b)



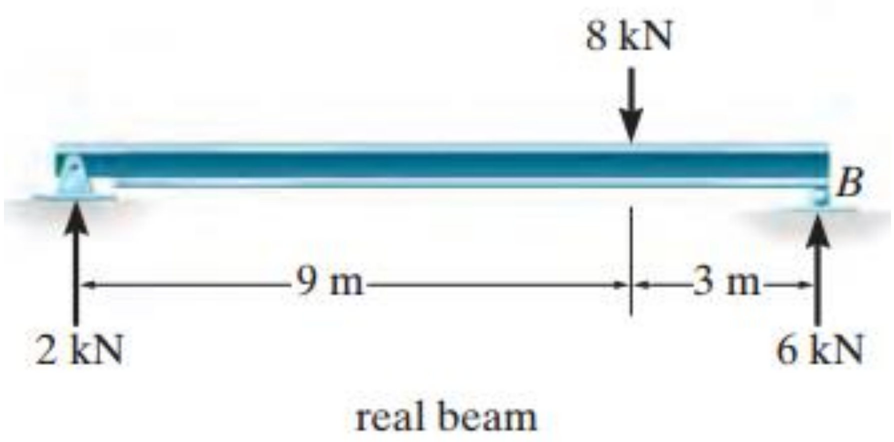
reactions
(c)

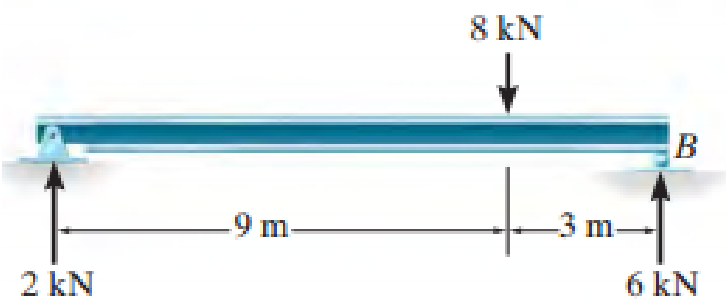
$$\theta_B = V_{B'} = -\frac{250}{EI}$$

$$\Delta_B = M_{B'} = -\frac{250}{EI}\left(5+\frac{10}{3}\right) = -\frac{6.250}{3EI}$$

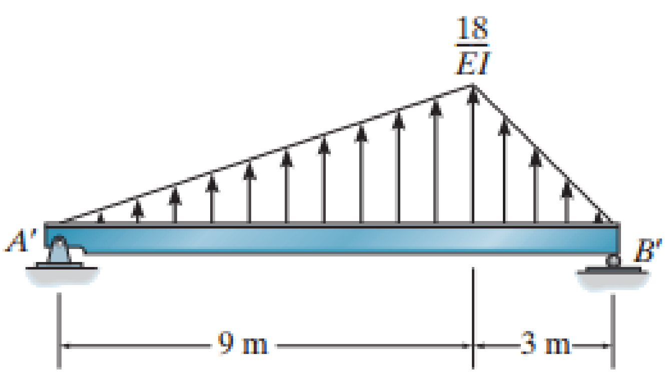
Example 2

- Determine the maximum deflection of the steel beam shown in figure.
- Use E = 200 GPa, I = $60(10^6)$ mm^4

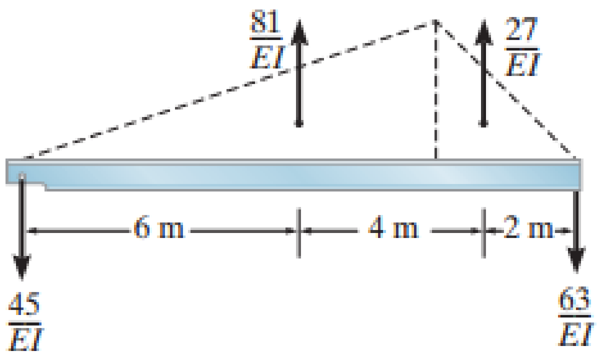




real beam

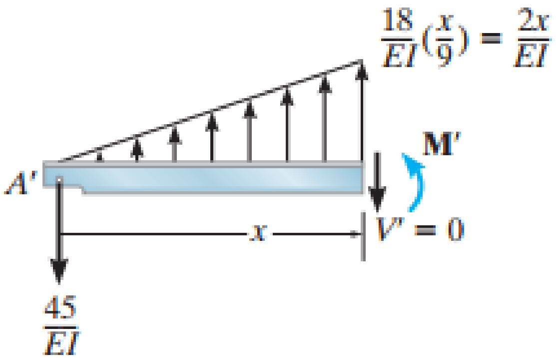


conjugate beam



external reactions

Maximum deflection of the real beam occurs at the point where the *slope* of the beam is *zero*. This corresponds to the same point in the conjugate beam where the *shear* is *zero*.



internal reactions

Assuming this point acts within the region from 0 ≤ x ≤ 9 m from A', we can isolate the section shown in Fig. Note that the peak of the distributed loading was determined from proportional triangles, that is, w/x = (18/EI)/9. We require V' = 0, so that :

$$+\uparrow \sum F_y = 0 \qquad -\frac{45}{EI} + \frac{1}{2}\left(\frac{2x}{EI}\right)x = 0 \quad \rightarrow x = 6{,}71\,\text{m}$$

$$\Delta_{\text{maks}} = M' = -\frac{45}{EI}(6{,}71) + \left[\frac{1}{2}\left(\frac{2\times 6{,}71}{EI}\right)6{,}71\right]\frac{1}{3}(6{,}71) = -\frac{201{,}2}{EI}\,\text{kN.m}^3$$

$$= -\frac{201{,}2}{200\times 60} = -0{,}0168\,\text{m}$$

Soal Latihan (Chapter VIII)

- **8.26**
- **8.27**
- **8.28**
- **8.29**
- **8.30**
- **8.31**
- **8.32**
- **8.33**
- **8.34**
- **8.35**
- **8.36**
- **8.37**
- **8.38**