11 VIBRATION AND VIBRATION ISOLATION

11.1 SIMPLE HARMONIC MOTION

Units of Vibration

In most vibration problems we are dealing with harmonic motion, where the quantities can be expressed as sine or cosine functions. The general formula for the harmonic displacement of a body is given by

$$x = X \sin \omega t \tag{11.1}$$

The velocity can be calculated by differentiating the displacement with respect to time

$$\dot{x} = \frac{dx}{dt} = X \omega \cos \omega t = V \sin (\omega t + \frac{\pi}{2}) \tag{11.2}$$

and the acceleration by differentiating the velocity

$$\ddot{x} = \frac{dv}{dt} = \frac{d^2 x}{dt^2} = -X \omega^2 \sin \omega t = -A \sin (\omega t) \tag{11.3}$$

These lead to simple relationships between the amplitudes

$$A = \omega V = \omega^2 X \tag{11.4}$$

Displacement, velocity, and acceleration are vector quantities that have a fixed angular relationship with each other, as the vector plot in Fig. 11.1 illustrates. Each vector rotates counterclockwise in time about the origin at the radial frequency, ω. Velocity leads displacement by 90° and acceleration leads displacement by 180°.

The units used in vibration measurements are more varied than those for sound level measurements. Amplitudes can be expressed in terms of displacement, velocity, acceleration, and jerk (the rate of change of acceleration). Accelerations are given not only in terms of length per time squared but also in terms of the standard gravitational acceleration, g. The peak amplitudes are simply coefficients such as those shown in Eq. 11.4. The root mean

FIGURE 11.1 Vector Representation of Harmonic Displacement, Velocity, and Acceleration

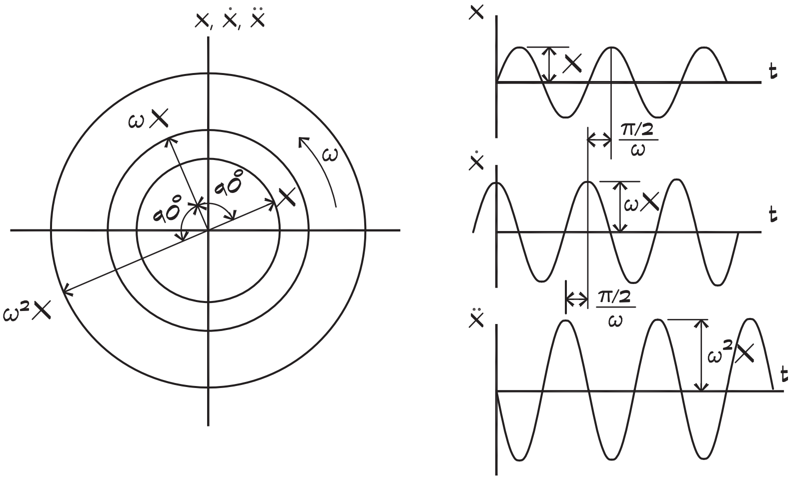


TABLE 11.1 Reference Quantities for Vibration Levels (Beranek and Ver, 1992)

Level (dB)	Formula	Reference (SI)
Acceleration	$L_a = 20 \log (a / a_o)$	$a_o = 10\ \mu m / s^2$
		$a_o = 10^{-5}\ m / s^2$
		$a_o = 1\ g$
		$a_o = 9.8\ m / s^2$
Velocity	$L_v = 20 \log (v / v_o)$	$v_o = 10\ n\,m / s$
		$v_o = 10^{-8}\ m / s$
Displacement	$L_d = 20 \log (d / d_o)$	$d_o = 10\ p\,m$
		$d_o = 10^{-11}\ m$

Note: Decimal multiples are 10^{-1} = deci (d), 10^{-2} = centi (c), 10^{-3} = milli (m), 10^{-6} = micro (μ), 10^{-9} = nano (*n*), and 10^{-12} = pico (*p*).

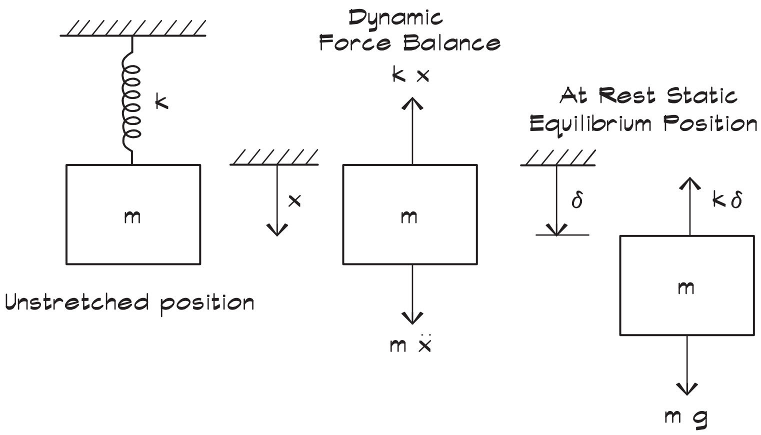
square (rms) value is the square root of the average of the square of a sine wave over a complete cycle, which is $\left(\sqrt{2}\right)^{-1}$ or .707 times the peak amplitude. Vibration amplitudes also can be expressed in decibels and Table 11.1 shows the preferred reference quantities.

11.2 SINGLE DEGREE OF FREEDOM SYSTEMS

Free Oscillators

In its simplest form a vibrating system can be represented as a spring mass, shown in Fig. 11.2. Such a system is said to have a single degree of freedom, since its motion can be described with a knowledge of only one variable, in this case its displacement.

FIGURE 11.2 Free Body Diagram of a Spring Mass System



In general if a system requires n numbers to describe its motion it is said to have n degrees of freedom. A completely free mass has six degrees of freedom: three orthogonal displacement directions and three rotations, one about each axis. A stretched string or a flexible beam has an infinite number of degrees of freedom, since there are an infinite number of possible vibration shapes. These can be analyzed in a regular manner using a superposition of all possible vibrational modes added together; however, to do so exactly requires an infinite number of constants, one for each mode. This mathematical construct, called a Fourier series, is a useful tool even if it is not carried out to infinity.

The forces on a simple spring mass system are the spring force, which depends on the displacement away from the equilibrium position, and the inertial force of the accelerating mass. The equation of motion was discussed in Chapt. 6 and is simply a summation of the forces on the body

$$\mathrm{m}\,\ddot{\mathrm{x}} + k\,\mathrm{x} = 0 \tag{11.5}$$

which has a general solution

$$\mathrm{x} = \mathrm{X}\,\sin\,(\omega_{\mathrm{n}}\,\mathrm{t} + \phi) \tag{11.6}$$

where $\omega_{\mathrm{n}} = \sqrt{k/\mathrm{m}}$ = undamped natural frequency (rad / s)
k = spring constant (N / m)
m = mass (kg)
ϕ = phase angle at time t = 0 (rad)
X = maximum displacement amplitude (m)

Although the spring mass model is simple, it is applicable as an approximation to many complicated structures. Building elements such as beams, wood or concrete floors, high-rise buildings, and towers can be modeled as spring mass systems and in more complex structures as series of connected elements, each having mass and stiffness.

Damped Oscillators

In vibrating systems, when bodies are set into motion, dissipative forces arise that damp or resist the movement. These are viscous forces that are proportional to the velocity of the body; however, not all types of damping are velocity dependent. Coulomb damping due to sliding friction, for example, is a constant force. To model viscous damping, such as that provided by a shock absorber, we refer to the spring mass system shown in Fig 11.3. Here the damping force is proportional to the velocity and is negative because the force opposes the direction of motion.

$$F_r = -c\,\dot{x} \tag{11.7}$$

where F_r = viscous damping force, (N)

c = resistance damping coefficient (N s / m)

$\dot{x} = \dfrac{dx}{dt}$ = first time derivative of the displacement

= velocity (m / s)

If we gather together all forces operating on the mass on the left-hand side, and equate it to the mass times the acceleration on the right-hand side in accordance with Newton's law, and rearrange the terms, we get

$$m\,\ddot{x} + c\,\dot{x} + k\,x = 0 \tag{11.8}$$

The general solution has the form $x = e^{a\,t}$, where a is a constant to be determined. Substituting into Eq. 11.8 we obtain

$$\left(a^2 + \frac{c}{m}a + \frac{k}{m}\right) e^{a\,t} = 0 \tag{11.9}$$

which holds for all t when

$$\left(a^2 + \frac{c}{m}a + \frac{k}{m}\right) = 0 \tag{11.10}$$

This equation, known as the characteristic equation, has two roots

$$a_{1,2} = -\frac{c}{2\,m} \pm \sqrt{\left(\frac{c}{2\,m}\right)^2 - \frac{k}{m}} \tag{11.11}$$

from which we can construct a general steady-state solution in the underdamped condition, where the term under the radical is negative.

$$x = X\,e^{\frac{-c\,t}{2\,m}} \sin\left(\omega_n\,t + \phi\right) \tag{11.12}$$

The damped natural frequency of vibration is given by

$$\omega_d = 2\pi\,f_d = \sqrt{\omega_n^2 - \left(\frac{c}{2\,m}\right)^2} \tag{11.13}$$

The damping coefficient, c, influences both the amplitude and the damped natural frequency of oscillation, ω_d, by slowing it down slightly.

An example of a damped oscillation is shown in Fig. 11.4. The envelope of the decay is controlled by the damping coefficient. One measure of the degree of damping is the decay

FIGURE 11.3 A Spring Mass System with Viscous Damping (Thomson, 1965)

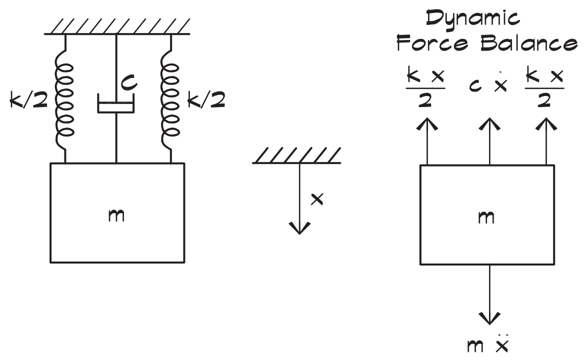
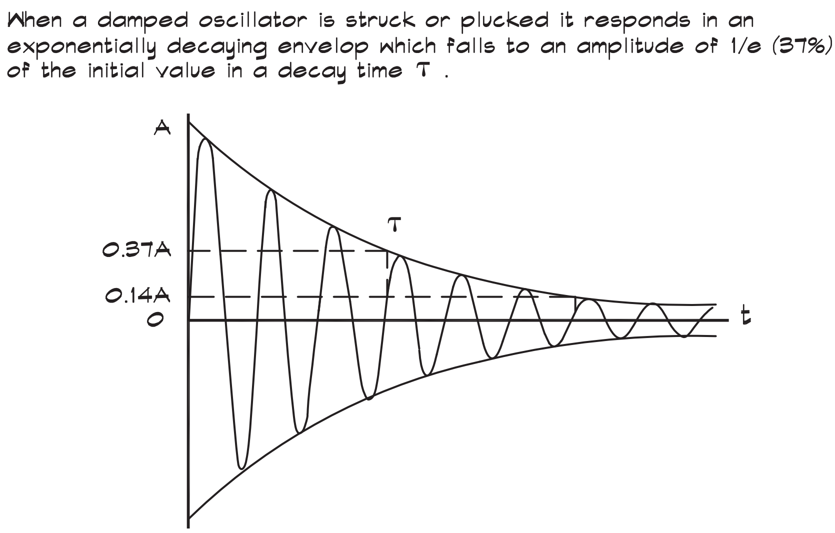


FIGURE 11.4 Response of a Damped Oscillator to an Impulse (Rossing and Fletcher,1995)



time, $\tau = \dfrac{2\,\mathrm{m}}{\mathrm{c}}$, which is the time it takes for the amplitude of the envelope to fall to 1/e (37%) of its initial value. It can be seen from Eq. 11.13 that, when one over the decay time is equal to the undamped natural frequency, the term under the radical is zero and the system does not oscillate. Such a system is said to be critically damped. The value of the damping coefficient at this point is given the symbol $c_c = 2\,\mathrm{m}\,\omega_{\mathrm{n}}$, and the degree of damping is expressed in terms of the ratio of the damping coefficient to the critical damping coefficient $\eta = \dfrac{c}{c_c}$, which is called the damping ratio, and is expressed as a percentage of critical damping.

Damping Properties of Materials

All materials have a certain amount of intrinsic internal damping, which depends on the internal structure of the substance. Figure 9.10 showed the damping coefficients for a number of common construction materials, which range from extremely low values in steel and other metals to very high values in resins and viscous liquids. These latter materials are used in laminated glass specifically for their damping characteristics. In laminated glass a resin is sandwiched between the two layers. This is called a constrained layer damper. Damping compounds are commercially available in bulk and can be trowelled directly onto lightweight metal panels. In order to be effective they should be applied thickly–to at least the thickness of the vibrating panel.

In wood floor systems panel adhesive can help provide damping when applied between sheets of flooring, between wood joists and plywood subfloors, and to stepped blocking installed within the floor joists. In concrete floor systems the thickness and density of the concrete determines the amount of damping. Additional damping can be provided by plates welded to the joist webs and by lightweight interior partitions attached either above or below the floor. Even if partitions are not load bearing, they can contribute significantly to damping.

Driven Oscillators and Resonance

When a spring mass system is driven by a periodic force, it will respond in a predictable manner, which depends on the frequency of the driving force. A familiar example is a child's swing. If a child pumps the swing by kicking his legs out at the proper moment, he can increase the amplitude of the swing oscillation. The swing responds at the frequency of the driving force but its amplitude increases substantially only when the period of the driving force matches the natural period of vibration. Thus the child soon learns that he must kick out his legs at the proper time if he is to increase his swing's height.

There are many examples of resonant systems in architecture, including sound waves in rectangular rooms, organ pipes, and other open or closed tubes; and structural systems including floors, walls and wall panels, piping, and mechanical equipment. Each of these can act as an oscillator and be driven into resonance by a periodic force. The equation describing the motion of a forced oscillator with damping is

$$\mathrm{m}\,\ddot{\mathrm{x}} + c\,\dot{\mathrm{x}} + k\,\mathrm{x} = \mathrm{F}_0 \sin(\omega\,\mathrm{t}) \tag{11.14}$$

The general solution has the form

$$\mathrm{x} = \mathrm{X} \sin(\omega\,\mathrm{t} - \phi) \tag{11.15}$$

By substituting into Eq. 11.14 we obtain

$$\begin{aligned} \mathrm{m}\,\omega^2\,\mathrm{X} \sin(\omega\,\mathrm{t} - \phi) - c\,\omega\,\mathrm{X} \sin\left(\omega\,\mathrm{t} - \phi + \frac{\pi}{2}\right) \\ - k\,\mathrm{X} \sin(\omega\,\mathrm{t} - \phi) + \mathrm{F}_0 \sin(\omega\,\mathrm{t}) = 0 \end{aligned} \tag{11.16}$$

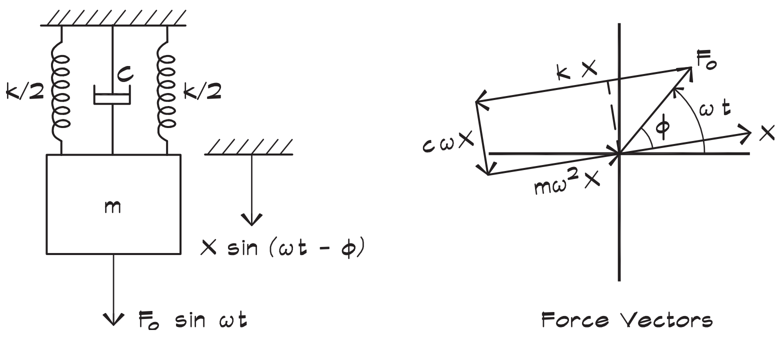
The relationship among all the forces acting on the mass is shown in Fig. 11.5, and from the geometry of the force triangle we can solve for the amplitude X

$$\mathrm{X} = \frac{\mathrm{F}_0}{\sqrt{\left(k - \mathrm{m}\,\omega^2\right)^2 + (c\,\omega)^2}} \tag{11.17}$$

and

$$\tan\phi = \frac{c\,\omega}{k - \mathrm{m}\,\omega^2} \tag{11.18}$$

FIGURE 11.5 Forced Response of a Spring Mass System with Viscous Damping (Thomson, 1965)



We can use more general notation as follows

$\omega_n = \sqrt{k/\mathrm{m}}$ = undamped natural frequency (rad / s)
$c_c = 2\ \mathrm{m}\ \omega_n$ = critical damping coefficient (N s / m)
$\eta = c/c_c$ = damping factor
$X_0 = F_0 / k$ = static deflection of the spring mass under the steady force F_0 (m)

and write Eq. 11.17 as

$$\frac{X}{X_0} = \frac{1}{\sqrt{\left[1 - (\omega/\omega_n)^2\right]^2 + \left[2\eta\,(\omega/\omega_n)\right]^2}} \tag{11.19}$$

and Eq. 11.18 as

$$\tan\phi = \frac{2\ \eta\ (\omega/\omega_n)}{1 - (\omega/\omega_n)^2} \tag{11.20}$$

Looking at Eqs. 11.15, 11.17, and 11.19 we see that the mass vibrates at the driving frequency ω, but the amplitude of vibration depends on the ratio of the squares of the resonant and driving frequencies. When the driving frequency matches the resonant frequency a maximum in the displacement occurs. Note that the damping term $2\ \eta\left(\omega/\omega_n\right)$ keeps the denominator from vanishing and limits the excursion at resonance.

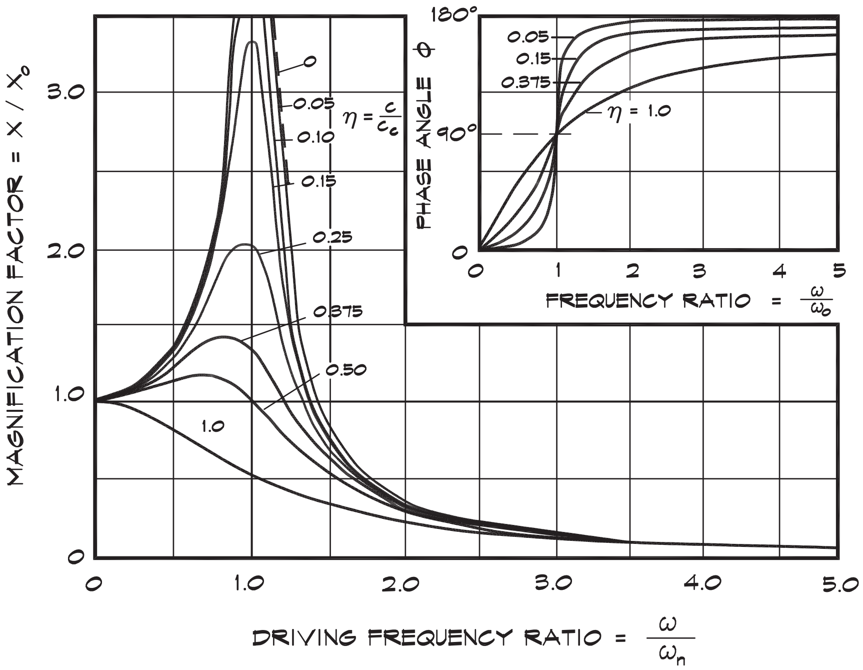
Figure 11.6 shows a plot of the response of the system. As the driving frequency moves toward the resonant frequency the output increases—theoretically reaching infinity at resonance for zero damping. The damping not only limits the maximum excursion at resonance but also shifts the resonant peak downward in frequency.

Vibration Isolation

When a simple harmonic force is applied to a spring mass system, it induces a response that reaches a maximum at the resonant frequency of the system. If we ask what force is transmitted to the foundation through the spring mass support we can refer again to Fig. 11.5.

The forces are transmitted to the support structure through the spring and shock absorber system. The formulas remain the same whether the mass is resting on springs or hung

FIGURE 11.6 Normalized Excursion vs Frequency for a Forced Simple Harmonic System with Damping (Thomson, 1965)



from springs. The balance of dynamic forces is shown, and using this geometry we can resolve the force on the support system as

$$F_t = \sqrt{(k\,X)^2 + (c\,\omega\,X)^2} = X\sqrt{k^2 + c^2\,\omega^2} \tag{11.21}$$

Using the expression given in Eq. 11.19 for the relationship between the applied force and the displacement amplitude, we can solve for the ratio of the impressed and transmitted forces

$$F_t = \frac{F_0\sqrt{1 + \left(\frac{c\,\omega}{k}\right)^2}}{\sqrt{\left[1 - \frac{m\,\omega^2}{k}\right]^2 + \left(\frac{c\,\omega}{k}\right)^2}} \tag{11.22}$$

which can be written as

$$\tau = \frac{F_t}{F_0} = \frac{\sqrt{1 + \left(2\,\eta\,\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\,\eta\,\frac{\omega}{\omega_n}\right)^2}} \tag{11.23}$$

FIGURE 11.7 Transmissibility of a Viscous Damped System
The Force Transmissibility and Motion Transmissibility of a Viscous Damped Single Degree of Freedom are Numerically Identical

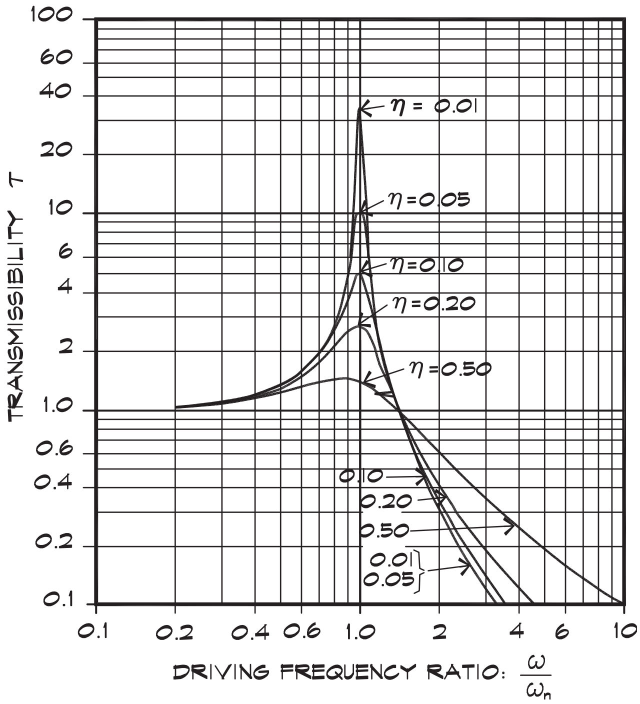


Figure 11.7 shows a plot of this expression in terms of the transmissibility, which is the ratio of the transmitted to the imposed force. We can see that above a given frequency $\left(\sqrt{2}\ f_n\right)$, as the frequency of the driving force increases, the transmissibility decreases and we achieve a decrease in the transmitted force. This is the fundamental principle behind vibration isolation.

Since the isolation is dependent on frequency ratio, the lower the resonant frequency, the greater the isolation for a given excitation frequency. The natural frequency of the spring mass system is

$$f_n = \frac{\omega_n}{2\,\pi} = \frac{1}{2\,\pi}\sqrt{k/m} = \frac{1}{2\,\pi}\sqrt{k\ g/m\ g} \tag{11.24}$$

which can be written in terms of the static deflection of the vibration isolator under the weight of the supported object,

$$f_n = \frac{1}{2\,\pi}\sqrt{g/\delta} = \frac{3.13}{\sqrt{\delta_i}}\ (\text{Hz},\ \delta_i \text{ in inches}) \tag{11.25}$$

or

$$f_n = \frac{1}{2\,\pi}\sqrt{g/\delta} = \frac{5}{\sqrt{\delta_{cm}}}\ (\text{Hz},\ \delta_{cm} \text{ in centimeters}) \tag{11.26}$$

A fundamental principle for effective isolation is that the greater the deflection of the isolator, the lower the resonant frequency of the spring mass system, and the greater the vibration isolation. We must counterbalance this against the mechanical stability of the isolated object since very soft mounts are generally less stable than stiff ones. To increase the deflection, we must increase the load on each isolator, so a few point-mount isolators are preferable to a continuous mat or sheet. Thick isolators are generally more effective than thin isolators since thick isolators can deflect more than thin ones. Finally, trapped air spaces under isolated objects should be avoided and, if unavoidable, then wide spaces are better than narrow spaces, because the trapped air acts like another spring. Note that the greater the damping, the less the vibration isolation, but the lower the vibration amplitude near resonance. This leads to a second important point, which is that damping is incorporated into vibration isolators, not to increase the isolation, but to limit the amplitude at resonance. An example might be a machine that starts from a standstill (zero frequency), goes through the isolator resonance, and onto its operating point frequency. If this happens slowly we may be willing to trade off isolation efficiency at the eventual operating point for amplitude limitation at resonance.

If there is zero damping Eq. 11.23 can be simplified further. Assuming that the frequency ratio is greater than $\sqrt{2}$, the transmissibility is given by

$$\tau \cong \left[\left(\frac{\omega}{\omega_{\mathrm{n}}} \right)^2 - 1 \right]^{-1} \tag{11.27}$$

We substitute $\omega_{\mathrm{n}}^2 = g/\delta$, where g is the acceleration due to gravity and δ is the static deflection of the spring under the load of the supported mass, and the transmissibility becomes

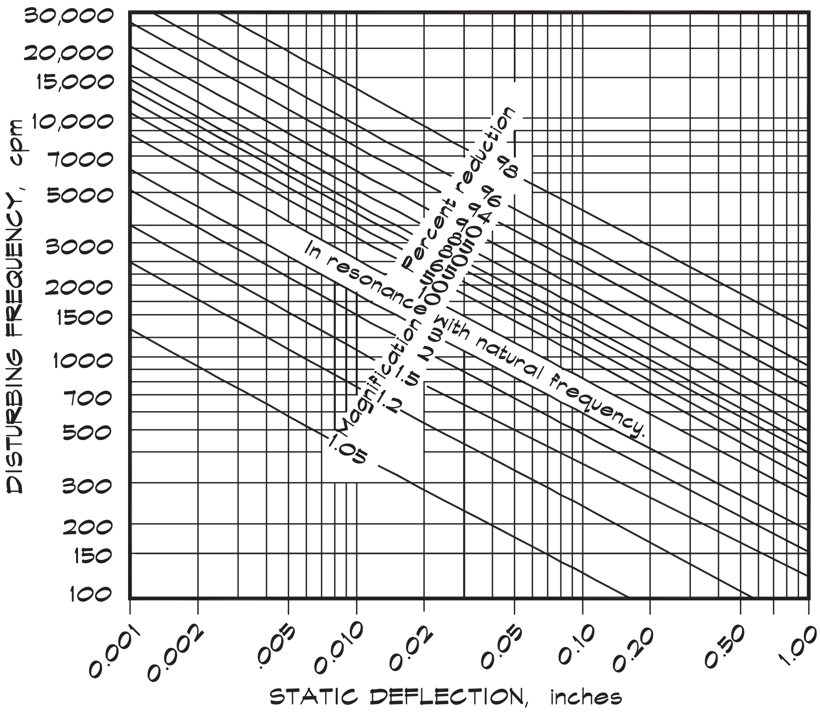
$$\tau \cong \left[\frac{(2\,\pi\,\mathrm{f})^2\,\delta}{\mathrm{g}} - 1 \right]^{-1} \tag{11.28}$$

which is sometimes expressed as an isolation efficiency or percent reduction in vibration in Fig. 11.8. This simplification is occasionally encountered in vibration isolation specifications that call for a given percentage of isolation at the operating point. It is better to specify the degree of isolation indirectly by calling out the deflection of the isolator, which is directly measurable by the installing contractor, rather than an efficiency that is abstract and difficult to measure in the field.

It is important to recall that these simple relationships only hold for single degree of freedom systems. If we are talking about a piece of mechanical equipment located on a slab the deflection of the slab under the weight of the isolated equipment must be very low—typically 8 to 10 times less than the deflection of the isolator for this approximation to hold. As the stiffness of the slab decreases, softer vibration isolators must be used to compensate.

When the excitation force is applied directly to the supported object or when it is self excited through eccentric motion, vibration isolators do not decrease the amplitude of the driven object but only the forces transmitted to the support system. When the supported object is excited by the motion of the support base, there is a similar reduction in the forces transmitted to the object. For a given directly applied excitation force, an inertial base consisting of a large mass, such as a concrete slab placed between the vibrating equipment and the support system, can decrease the amplitude of the supported equipment, but interestingly

FIGURE 11.8 Isolation Efficiency for a Flexible Mount



not the amplitude of the transmitted force. Inertial bases are very helpful in attenuating the motion of mechanical equipment such as pumps, large compressors, and fans, which can have eccentric loads that are large compared to their intrinsic mass.

Isolation of Sensitive Equipment

Frequently there are requirements to isolate a piece of sensitive equipment from floor-induced vibrations. The geometry is that shown in Fig. 11.9. Since the spring supports are in their linear region the relations are the same for equipment hung from above or supported

FIGURE 11.9 Force Vectors of a Spring Mass System with Viscous Damping for a Moving Support

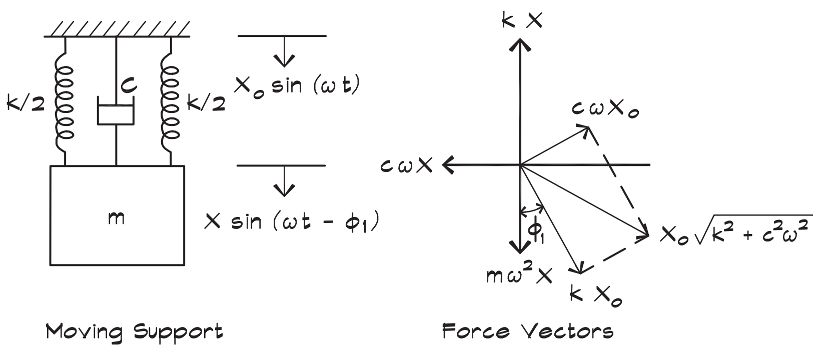
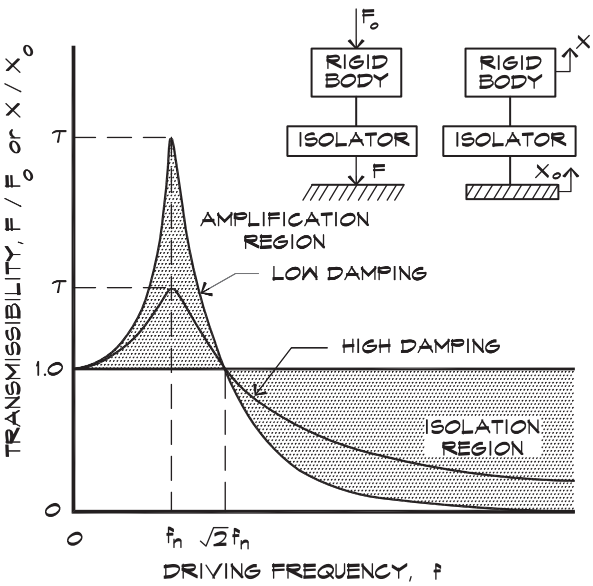


FIGURE 11.10 Transmissibility Curves for Vibration Isolation (Ruzicka, 1971)



from below. The transmissibility is the same as that given in Eq. 11.23. In the case of isolated equipment, instead of the force being generated by a vibrating machine, a displacement is created by the motion of the supporting foundation. In Eq. 11.23 the terms for force amplitudes are replaced by displacement amplitudes.

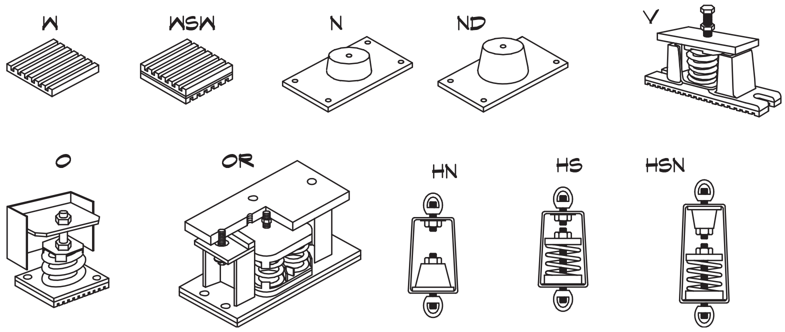
Summary of the Principles of Isolation

Figure 11.10 shows the result of this analysis for both self-excited sources and sensitive receivers. The transmission equation is the same in both cases, differing only in the definition of transmissibility, which for an imposed driving force is the force ratio and for base motion is the displacement ratio. Above the resonant frequency of the spring mass system the response to the driving function decreases until, at a frequency just over 40% above resonance, the response amplitude is less than the imposed amplitude. At higher driving frequencies the response is further decreased. The lower the natural frequency of the isolator—that is, the greater its deflection under the load of the equipment—the greater the isolation.

11.3 VIBRATION ISOLATORS

Commercially available vibration isolators fall into several general categories: resilient pads, neoprene mounts, and a combination of a steel spring and neoprene pad (Fig. 11.11). An isolator is listed by the manufacturer with a range of rated loads and a static deflection, which is the deflection under the maximum rated load. Most isolators will tolerate some loading beyond their rated capacity, often as much as 50%; however, it is good practice to check the published load versus deflection curve to be sure. An isolator must be sufficiently loaded to achieve its rated deflection, but it must also remain in the linear range of the load versus deflection curve and not bottom out.

FIGURE 11.11 Types of Vibration Isolators



Isolation Pads (Type W, WSW)

Isolation pads of felt, cork, neoprene impregnated fiberglass, or ribbed neoprene sometimes sandwiched by steel plates usually have about a .05 inch (1 mm) deflection ($f_n = 14$ Hz) and are used in noncritical or high-frequency applications. Typically these products are supplied in small squares, which are placed under vibrating equipment or piping. Depending on the stiffness of the product, they are designed to be loaded to a particular weight per unit area of pad. For 40 durometer neoprene pads, for example, the usual load recommendation is about 50 lbs/sq in. Where higher deflections are desired or where there is a need to spread the load, pads are sandwiched with thin steel plates. Such pads are designated WSW or WSWSW depending on the number of pads and plates.

Neoprene Mounts (Type N, ND)

Neoprene isolators are available in the form of individual mounts, which have about a 0.25 inch (6 mm) rated deflection, or as double deflection mounts having a 0.4 inch (10 mm) deflection. These products frequently have integral steel plates, sometimes with tapped holes, that allow them to be bolted to walls or floors. They are available in neoprene of various durometers from 30 to 60, and are color-coded for ease of identification in the field. The double deflection isolators can be used to support floating floors in critical applications such as recording studios.

Steel Springs (Type V, O, OR)

A steel spring is the most commonly used vibration isolator for large equipment. Steel springs alone can be effective for low-frequency isolation; however, for broadband isolation they must be used in combination with neoprene pads to stop high frequencies. Otherwise these vibrations will be transmitted down the spring. Springs having up to 5 inches (13 cm) static deflection are available, but it is unusual to see deflections greater than 3 inches (8 cm) due to their lateral instability. Unhoused open-spring mounts (Type O) must have a large enough diameter (at least 0.8 times the compressed height) to provide a lateral stiffness equal to the vertical stiffness. Housed springs have the advantage of providing a stop for lateral (Type V) or vertical motion and an integral support (Type OR) for installing the equipment at or near its eventual height, but are more prone to ground out when improperly positioned. These stops are useful during the installation process since the load of the equipment or piping may vary; particularly if it can be filled with water or oil. Built in limit stops are not the same as earthquake restraints, which must resist motion in any direction. Threaded rods, allowing the height of the equipment to be adjusted and locked into place with double nuts, are also part of the isolator assembly.

Spring isolators must be loaded sufficiently to produce the design deflection, but not so much that the springs bottom out coil to coil. A properly isolated piece of equipment will move freely if one stands on the base, and should not be shorted out by solid electrical or plumbing connections.

Hanger Isolators (Type HN, HS, HSN)

Hanger isolators contain a flexible element, either neoprene (Type HN) or a steel spring (Type HS), or a combination of the two (Type HSN), which supports equipment from above. Spring hangers, like free standing springs, must have a neoprene pad as part of the assembly. Hangers should allow for some misalignment between the housing and the support rod (30°) without shorting out and be free to rotate 360° without making contact with another object. Threaded height-adjusting rods are usually part of these devices.

Air Mounts (AS)

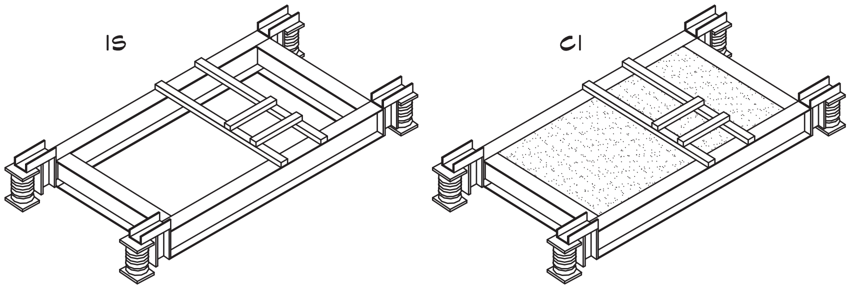
Air springs consisting of a neoprene bladder filled with compressed air are also available. These have the disadvantage of requiring an air source to maintain adequate pressure along with periodic maintenance to assure that there is no leakage. The advantage is that they allow easy level adjustment and can provide larger static deflections than spring isolators for critical applications.

Support Frames (Type IS, CI, R)

Since the lower the natural frequency of vibration the greater the vibration isolation, it is advantageous to maximize the deflection of the isolation system consistent with constraints imposed by stability requirements. If the support system is a neoprene mount—for example, under a vibrating object of a given mass—it is generally best to use the fewest number of isolators possible consistent with other constraints. It is less effective to use a continuous sheet of neoprene, cork, flexible mesh, or other similar material to isolate a piece of equipment or floating floor since the load per unit area and thus the isolator deflection is relatively low. Rather, it is better to space the mounts under the isolated equipment so that the load on each mount is maximized and the lowest possible natural frequency is obtained. A structural frame may have to be used to support the load of the equipment if its internal frame is not sufficient to take a point load. Integral steel (IS) or concrete inertial (CI) or rail frame (R) bases (Fig. 11.12) are used in these cases. A height-saving bracket that lowers the bottom of the frame to 25 to 50 mm (1" to 2") above the floor is typically part of an IS or CI frame. Brackets allow the frame to be placed on the floor and the equipment mounted to it before the springs are slid into place and adjusted.

When equipment is mounted on isolators the load is more concentrated than with equipment set directly on a floor. The structure beneath the isolators must be capable of supporting the point load and may require a 100 to 150 mm (4" to 6") housekeeping pad to help spread the load. Equipment such as small packaged air handlers mounted on a lightweight roof can be supported on built up platforms that incorporate a thin (3") concrete pad. Lighter platforms may be used if they are located directly above heavy structural elements such as steel beams or columns. In all cases the ratio of structural deflection to spring deflection must be less than 1:8 under the equipment load.

FIGURE 11.12 Vibration Isolation Bases



Isolator Selection

A number of manufacturers, as well as ASHRAE, publish recommendations on the selection of vibration isolators. By and large these recommendations assume that the building structure consists of concrete slabs having a given span between columns. One of the most useful is that published by Vibron Ltd. (Allen, 1989). This particular guide is reproduced as Tables 11.2 through 11.4. To use it, first determine the sensitivity of the receiving space, the floor thickness, and span. The longer the span, the more the deflection of the floor, the lower its resonant frequency, and the harder it is to isolate mechanical equipment that it supports. From step one we obtain an isolation category, a number from 1 to 6, which is a measure of the difficulty of successfully isolating the equipment. We then enter the charts in Tables 11.3 or 11.4 and pick out the base type and isolator deflection appropriate to the type of equipment and the isolation category.

When a concrete inertial (type CI) base is required, we can calculate its thickness from the nomographs given in Fig. 11.13. Using such a table is a practical way of selecting an appropriate isolator for a given situation. Although these tabular design methods are simple in practice, there is a great deal of calculating and experience that goes into their creation.

11.4 SUPPORT OF VIBRATING EQUIPMENT

Structural Support

A spring mass system, used to isolate vibrating equipment from its support structure, is based on a theory that assumes that the support system is very stiff. In practice it is important to construct support systems that are stiff, compared to the deflection of the isolators, and to minimize radiation from lightweight diaphragms. Where the support structure is very light—which can be the case for roof-mounted units—mechanical equipment is best supported on a separate system of steel beams that in turn are supported on columns down to a footing. A lightweight roof or similar structure can radiate sound like a driven loudspeaker, so mechanical equipment should not be located directly on lightweight roof panels. Where there is no other choice, and the roof slab is less than 4.5” (11 cm) of concrete, a localized concrete housekeeping pad should be used, having a thickness of 4” (10 cm) to 6” (15 cm) and a length 12” (30 cm) longer and wider than the supported equipment. These pads help spread the load and provide some inertial mass to increase the impedance of the support. Where it is not possible to locate equipment above a column, it should be located over one or more heavy structural members. Where supporting structures are less than 3.5” of solid concrete, use one isolation category above that determined from Table 11.2 along with the concrete subbase.

TABLE 11.2 Vibration Isolation Selection Guide (Vibron, 1989)

LOCATION OF VIBRATION ISOLATION		ISOLATION CATEGORY		
			UPPER CONCRETE FLOORS - OVER AND ADJACENT TO OCCUPIED AREAS	
TYPE OF BUILDING	FLOOR SPAN (ft)	BASEMENT BELOW GRADE	6" THICK & UP	TO 6" THICK
AUDITORIUMS CHURCHES - CRITICAL AREAS HOSPITALS - CRITICAL AREAS MUSIC HALLS OFFICE BUILDINGS - CRITICAL CONDOMINUIMS RADIO AND TV STATIONS	ON GRADE - ALL SPANS	—	2	2
	THROUGH 20	1	2	3
	21 - 30	1	3	4
	31 - 40	1	4	5
	41 - 50	1	5	6
	51 & UP	1	6	6
APARTMENT BUILDINGS CHURCHES HOSPITALS HOTELS OFFICE BUILDINGS PUBLIC BUILDINGS SCHOOLS	ON GRADE - ALL SPANS	—	2	2
	THROUGH 20	1	2	3
	21 - 30	1	3	3
	31 - 40	1	3	4
	41 - 50	1	4	5
	51 & UP	1	5	6
INDUSTRIAL BUILDINGS - NEAR OFFICE AREAS SPORTS ACTIVITY AREAS - INDOORS RESTAURANTS TRANSPORTATION - AIRPORTS AND BUS TERMINALS	ON GRADE - ALL SPANS	—	2	2
	THROUGH 20	1	2	2
	21 - 30	1	2	3
	31 - 40	1	3	3
	41 - 50	1	3	5
	51 & UP	1	4	5

TYPES OF ISOLATORS AND BASES USED IN THE TABLES

W	3/8" THICK, 40 DUROMETER RIBBED NEOPRENE PADS
WSW	3/8" THICK, 40 DUROMETER RIBBED NEOPRENE PADS SEPARATED BY A 10 GAUGE STEEL SHEET - LOADED TO 50 PSI
N	NEOPRENE IN SHEAR MOUNTS HAVING A 0.25" RATED DEFLECTION WITH INTEGRAL STEEL PLATES AND TAPPED HOLES FOR BOLTING EQUIPMENT
ND	DOUBLE DEFLECTION NEOPRENE MOUNTS HAVING A 0.4" RATED DEFLECTION WITH STEEL PLATES AND TAPPED HOLES FOR BOLTING EQUIPMENT
V	HOUSED STEEL SPRING AND 3/8" NEOPRENE ISOLATORS WITH RUBBER SNUBBERS SEPARATING THE TOP AND BOTTOM HOUSINGS
O	OPEN FREE STANDING STEEL SPRING AND 3/8" NEOPRENE ISOLATORS WITH EQUAL STIFFNESS IN THE HORIZONTAL AND VERTICAL DIRECTIONS
OR	OPEN RESTRICTED STEEL SPRING AND 3/8" NEOPRENE ISOLATORS WITH EQUAL STIFFNESS IN THE HORIZONTAL AND VERTICAL DIRECTIONS AND LIMIT STOPS TO PREVENT LIFTING AND STRESSING OF EQUIPMENT
HN	NEOPRENE MOUNT HANGERS FOR SUSPENDED EQUIPMENT - HANGERS SHALL ACCOMMODATE A 30 DEG. MISALIGNMENT WITHOUT SHORTING OUT
HS	SPRING AND NEOPRENE PAD HANGERS FOR SUSPENDED EQUIPMENT HANGERS SHALL ACCOMMODATE A 30 DEG. MISALIGNMENT WITHOUT SHORTING OUT
HSN	SPRING AND NEOPRENE MOUNT HANGERS FOR SUSPENDED EQUIPMENT HANGERS SHALL ACCOMMODATE A 30 DEG. MISALIGNMENT WITHOUT SHORTING OUT
R	SPRING MOUNT SYSTEM WHICH INCORPORATES AN INTEGRAL RAIL FRAME
IS	INTEGRAL STEEL FRAME DESIGNED TO SUPPORT THE EQUIPMENT AND MOTOR
CI	CONCRETE INERTIAL BASE WITH FULL DEPTH PERIMETER STEEL FRAMES

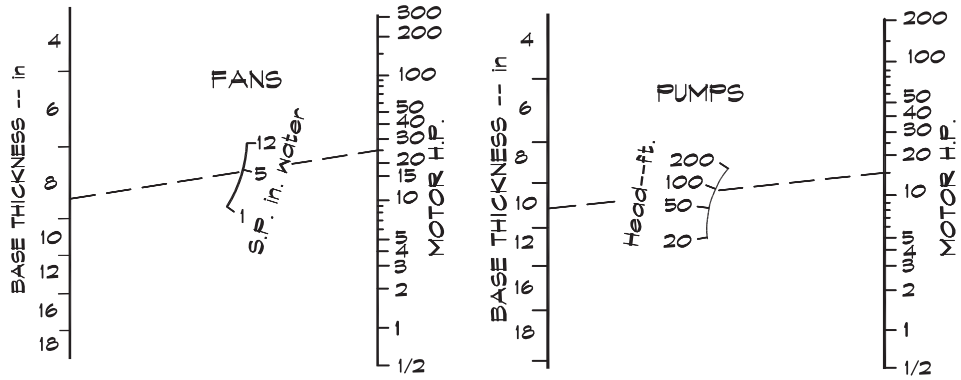
TABLE 11.3 Vibration Isolation Selection Guide (Vibron, 1989)

TYPE OF EQUIPMENT			VIBRATION ISOLATION CATEGORY																	
			1			2			3			4			5			6		
DESIGNATION	DESCRIPTION		BASE	ISL TR	DEFL	BASE	ISL TR	DEFL	BASE	ISL TR	DEFL	BASE	ISL TR	DEFL	BASE	ISL TR	DEFL	BASE	ISL TR	DEFL
CENTRIFUGAL FANS	THROUGH 40 HP	TO 275 RPM	IS	O	3.0	IS	O	3.0	IS	O	3.0	IS	O	3.0	IS	O	3.0	IS	O	3.0
		276-600 RPM	IS	O	1.5	IS	O	1.75	IS	O	2.0	IS	O	2.0	IS	O	2.0	IS	O	2.5
		601 RPM & UP	IS	O	1.0	IS	O	1.0	IS	O	1.0	IS	O	1.75	IS	O	1.75	IS	O	2.0
	50 HP & UP	TO 275 RPM	CI	O	3.0	CI	O	3.0	CI	O	3.5	CI	O	4.0	CI	O	4.0	CI	O	4.0
		276-600 RPM	CI	O	1.5	CI	O	1.75	CI	O	2.0	CI	O	2.5	CI	O	3.0	CI	O	3.0
		601 RPM & UP	CI	O	1.0	CI	O	1.0	CI	O	1.0	CI	O	1.75	CI	O	2.0	CI	O	3.0
TUBULAR CENTRIFUGAL AND AXIAL FANS	ALL SIZES SUSPENDED	TO 750 RPM	-	HS	1.0	-	HS	1.1	-	O	1.5	-	O	2.0	-	O	2.75	-	O	3.0
		751 RPM & UP	-	HS	0.7	-	HS	1.0	-	O	1.1	-	O	1.5	-	O	2.0	-	O	2.75
	ALL SIZES FLOOR MOUNTED	TO 750 RPM	IS	O	0.7	IS	O	1.1	IS	O	1.5	CI	O	2.0	CI	O	2.75	CI	O	3.0
		751 RPM & UP	IS	O	0.7	IS	O	1.0	IS	O	1.1	CI	O	1.5	CI	O	2.0	CI	O	2.75
PACKAGED AH, AC AND H & V UNITS (SUSPENDED)	THRU 7.5 HP	ALL RPM	-	HS	0.7	-	HS	0.7	-	HS	0.7	-	HS	1.0	-	HSN	1.0	-	HSN	1.0
	10 HP & UP AND TO 5" S.P.	TO 600 RPM	-	HS	0.7	-	HS	0.7	-	HS	1.0	-	HS	1.0	-	HSN	1.5	-	HSN	1.75
		601-1000 RPM	-	HS	0.7	-	HS	0.7	-	HS	1.0	-	HS	1.0	-	HSN	1.0	-	HSN	1.5
		1001 RPM & UP	-	HS	0.7	-	HS	0.7	-	HS	0.7	-	HS	0.7	-	HSN	1.0	-	HSN	1.0
PACKAGED AH, AC AND H & V UNITS (FLOOR MOUNTED)	THRU 7.5 HP	ALL RPM	-	ND	0.25	-	V	0.75	-	V	0.75	-	V	1.0	-	V	1.0	-	V	1.0
	10 HP & UP AND TO 5" S.P.	TO 600 RPM	-	V	0.7	-	V	0.7	-	V	1.0	-	O	1.0	-	O	1.5	-	O	1.75
		601-1000 RPM	-	V	0.7	-	V	0.7	-	V	1.0	-	O	1.0	-	O	1.0	-	O	1.5
		1001 RPM & UP	-	V	0.7	-	V	0.7	-	V	0.7	-	O	1.0	-	O	1.0	-	O	1.0
UTILITY SETS	SUSPENDED	TO 500 RPM	-	HS	0.35	-	HS	1.0	-	HS	1.0	-	HS	1.5	-	HS	2.0	-	HS	2.5
		501 RPM & UP	-	HS	0.35	-	HS	1.0	-	HS	1.0	-	HS	1.0	-	HS	1.5	-	HS	2.0
	FLOOR MOUNTED	TO 500 RPM	R	ND	0.35	IS	O	1.0	IS	O	1.5	IS	O	2.0	IS	O	2.5	IS	O	2.75
		501 RPM & UP	R	ND	0.35	IS	O	1.0	IS	O	1.0	IS	O	1.5	IS	O	2.0	IS	O	2.5
AIR COOLED CONDENSERS AND CLOSED CIRCUIT COOLING TOWERS COOLERS	CENTRIFUGAL UNITS	TO 400 RPM	-	OR	2.0	-	OR	2.0	-	OR	2.0	-	OR	2.5	-	OR	3.0	-	OR	3.0
		401 RPM & UP	-	OR	1.0	-	OR	1.0	-	OR	1.5	-	OR	1.5	-	OR	2.0	-	OR	2.0
	AXIAL FLOW UNITS	TO 300 RPM	-	OR	2.0	-	OR	2.0	-	OR	2.0	-	OR	3.0	-	OR	3.0	-	OR	3.5
		301-700 RPM	-	OR	1.5	-	OR	1.5	-	OR	1.5	-	OR	2.0	-	OR	2.0	-	OR	3.0
		701 RPM & UP	-	OR	1.0	-	OR	1.0	-	OR	1.5	-	OR	1.5	-	OR	2.0	-	OR	3.0
PACKAGED STEAM GENERATORS	ALL SIZES		-	OR	1.0	-	OR	1.5	-	OR	1.75	-	OR	2.0	-	OR	2.5	-	OR	2.5

TABLE 11.4 Vibration Isolation Selection Guide (Vibron, 1989)

TYPE OF EQUIPMENT			VIBRATION ISOLATION CATEGORY																	
			1			2			3			4			5			6		
DESIGNATION	DESCRIPTION		BASE	ISLTR	DEFL	BASE	ISLTR	DEFL	BASE	ISLTR	DEFL	BASE	ISLTR	DEFL	BASE	ISLTR	DEFL	BASE	ISLTR	DEFL
CENTRIFUGAL PUMPS	CLOSE COUPLED ALL RPM	THRU 5 HP	CI	O	1.0	CI	O	1.0	CI	O	1.0	CI	O	1.0	CI	O	1.5	CI	O	1.5
		7.5 HP & UP	CI	O	1.0	CI	O	1.0	CI	O	1.0	CI	O	1.5	CI	O	2.0	CI	O	2.0
	BASE MOUNTED ALL RPM	THRU 5 HP	CI	O	1.0	CI	O	1.0	CI	O	1.0	CI	O	1.5	CI	O	2.0	CI	O	2.0
		7.5 HP - 75 HP	CI	O	1.0	CI	O	1.0	CI	O	1.0	CI	O	1.5	CI	O	2.0	CI	O	3.5
		75 HP & UP	CI	O	1.0	CI	O	1.0	CI	O	1.5	CI	O	2.0	CI	O	2.5	CI	O	3.0
INLINE PUMPS	THRU 40 HP		-	WSW	0.15	-	V	0.7	CI	O	1.0	-	O	1.0	CI	O	1.5	CI	O	2.0
	40 HP & UP		-	WSW	0.15	-	V	1.0	CI	O	1.0	CI	O	1.5	CI	O	2.0	CI	O	2.5
RECIPROCATING AIR OR REFRIGERATION COMPRESSORS	THRU 15 HP	TO 500 RPM	CI	O	1.0	CI	O	1.0	CI	O	1.5	CI	O	1.75	CI	O	2.0	CI	O	2.5
		501-750 RPM	CI	O	0.75	CI	O	1.0	CI	O	1.5	CI	O	1.5	CI	O	2.0	CI	O	2.0
		751 RPM & UP	CI	O	0.75	CI	O	1.0	CI	O	1.5	CI	O	1.5	CI	O	2.0	CI	O	2.0
	20 HP AND UP	TO 500 RPM	CI	O	1.0	CI	O	1.5	CI	O	1.75	CI	O	2.0	CI	O	2.0	CI	O	1.75
		501-750 RPM	CI	O	1.0	CI	O	1.25	CI	O	1.5	CI	O	1.75	CI	O	1.75	CI	O	1.5
		751 RPM & UP	CI	O	0.75	CI	O	1.0	CI	O	1.25	CI	O	1.5	CI	O	1.5	CI	O	1.0
CENTRIFUGAL & REFRIGERATION MACHINES	ALL SIZES		IS	OR	1.0	IS	OR	1.5	IS	OR	2.0	IS	OR	2.0	IS	OR	2.5	IS	OR	2.5
ABSORPTION REFRIGERATION MACHINES	ALL SIZES		IS	OR	1.0	IS	OR	1.0	IS	OR	1.5	IS	OR	1.5	IS	OR	2.0	IS	OR	2.0
ELECTRICAL DISTRIBUTION TRANSFORMERS	0 - 50 KVA		-	W	0.06	-	WSW	0.12	-	WSW	0.12	-	WSW	0.12	-	V	0.7	-	V	0.7
	51 - 250 KVA		-	N	0.25	-	N	0.25	-	ND	0.35	-	ND	0.4	-	ND	0.4	-	ND	0.4
	251 KVA & LARGER		-	ND	0.35	-	V	0.7	-	V	0.7	-	V	0.7	-	V	1.0	-	V	1.0

FIGURE 11.13 The Thickness of Concrete Inertial Bases (Vibron, 1989)



For T-shaped Pump Bases Reduce To Next Lower Thickness

AIR AND REFRIGERATION COMPRESSORS	
Motor H.P.	Base Thickness -- in.
through 2	8
3 & 5	10
71/2 to 20	12
25 to 40	14
50 to 75	16
100& 125	20
125 up	24

Examples of various recommendations on the support of rooftop equipment are shown in Fig. 11.14 (Schaffer, 1991).

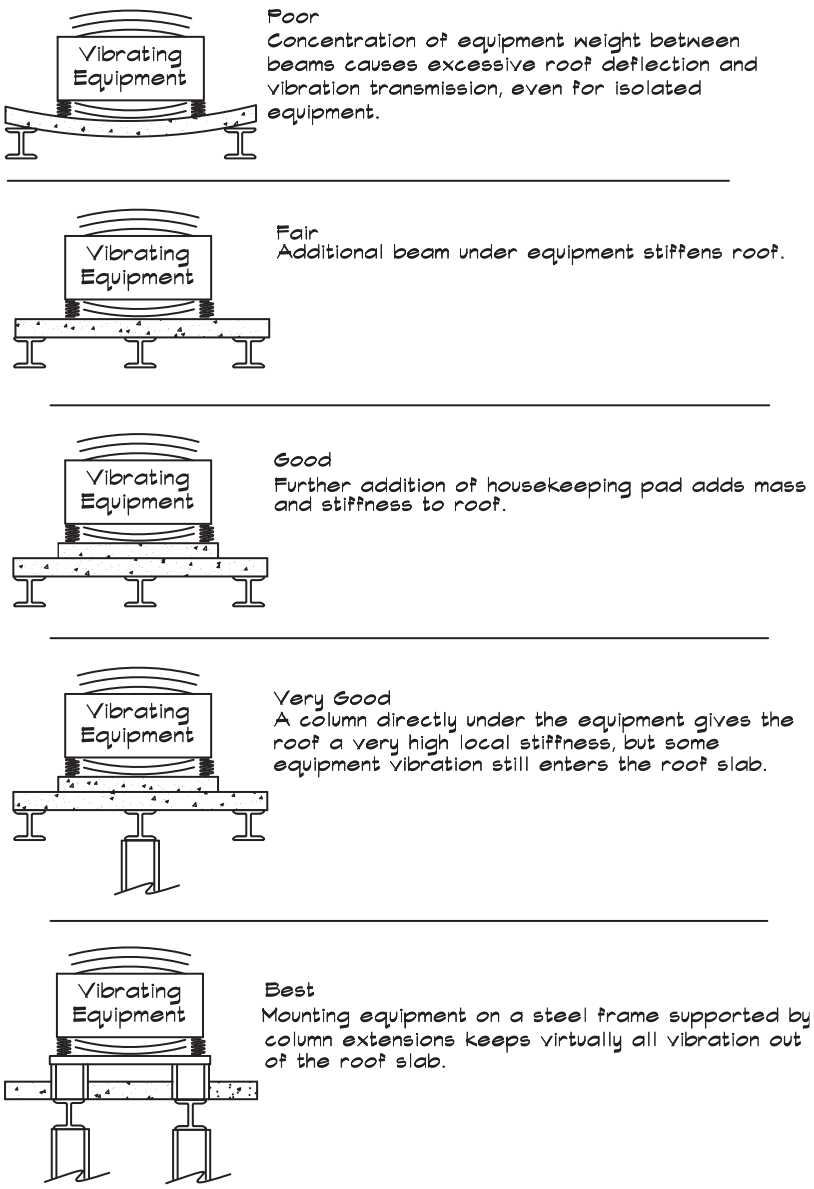
Inertial Bases

When the source of vibration is a piece of mechanical equipment with a large rotating mass or a high initial torque, it is good practice to mount it on a concrete base that is itself supported on spring isolators. The additional mass does not increase the isolation efficiency since the springs must be selected to support both the equipment and the base, and the overall spring deflection will probably not change appreciably. The advantage of having the base is that for a given driving force, such as the eccentricity of a rotating part, there is a lower overall displacement due to the extra mass of the combined base plus equipment. Inertial bases also aid in the stabilization of tall pieces of equipment, equipment with a large rocking component, and equipment requiring thrust restraint.

Concrete inertial bases are used in the isolation of pumps and provide additional frame stiffness, which a pump frequently requires. Pump bases are sized so that their weight is about two to three times that of the supported equipment. Any piping, attached to a pump mounted on an isolated base, must be supported from the inertial base or by overhead spring hangers. It must not be rigidly supported from a wall, floor, or roof slab unless it is in a noncritical location.

Where unbalanced equipment, such as single- or double-cylinder low-speed air compressors are to be isolated, the weight of the inertial base is calculated from the unbalanced

FIGURE 11.14 Structural Support of Rooftop Equipment (Schaffer, 1991)



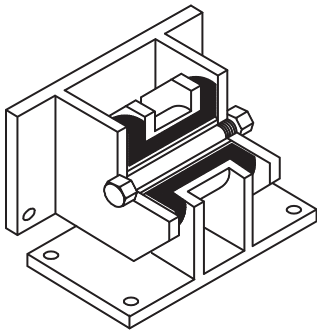
force, which can be obtained from the manufacturer. These bases frequently must be five to seven times the weight of the compressor to control the motion.

Concrete bases also offer resistance to induced forces such as fan thrust. Isolation manufacturers (Mason, 1968) recommend that a base weighing from one to three times the fan weight be used to control thrust for fans above 6” of static pressure.

Earthquake Restraints

In areas of high seismic activity, vibration isolated equipment must be constrained from moving during an earthquake. The seismic restraint system must not degrade the performance

FIGURE 11.15 Earthquake Restraint (Mason Industries, 1998)



of the vibration isolation. Some specialized isolators incorporate seismic restraints, but most vibration isolators do not since a restraint device must control motion in any direction. A standard method of providing three-dimensional restraint is shown in Fig. 11.15 using a commercial three-axis restraint system. Lightweight hanger-supported equipment can be restrained by means of several slack braided-steel cables. Any earthquake restraint system must comply with local codes and should be reviewed by a structural engineer.

Pipe Isolation

Piping can conduct noise and vibration generated through fluid motion and by being connected to vibrating equipment. Fluid flow in piping generates sound power levels that are dependent on the flow velocity. Pipes and electrical conduits that are attached directly to vibrating equipment and to a supporting structure serve as a transmission path, which short circuits otherwise adequate vibration isolation. Any rigid piping attached to isolated equipment such as pumps, refrigeration machines, and condensers must be separately vibration isolated, typically at the first three points of support, which for large pipe is about 15 m (50 ft). It should be suspended by means of an isolator having a deflection that is at least that of the supported equipment or 3/4", whichever is greater.

There is a significant difference in the weight of a large water pipe, depending on whether it is empty or filled. Isolated equipment will move up when the pipe system is drained, and in doing so, will stress elbows and joints. The suspension system should allow for normal motion of the pipe under these conditions. Risers and other long pipe runs will expand and contract as they are heated and cooled and should be resiliently mounted. Even when fluid is not flowing, a popping noise can be generated as the pipe slides past a stud or other support point during heating or cooling.

In critical applications such as condominiums, water, waste, and refrigeration pipes should be isolated from making contact with structural elements for their entire length. Table 11.5 gives typical recommendations on the types of materials used for the isolation of plumbing and piping. These recommendations also apply to the support of piping at points where it penetrates a floor.

Several examples of proper isolation of piping connected to pumps are shown in Fig. 11.16. On all piping greater than 5" (13 cm) diameter, flexible pipe couplings are necessary between the pump outlet and the pipe run. Even with smaller diameter pipes they can be very helpful in decreasing downstream vibrations and associated noise. They act as vibration isolators by breaking the mechanical coupling between the pump and the pipe, and they

TABLE 11.5 Typical Plumbing Isolation Materials

NOMINAL PIPE SIZE (Dia, in)	REQUIRED ISOLATION HORIZONTAL	REQUIRED ISOLATION VERTICAL
1/2	1/4" FELT	1/4" FELT
3/4	3/8" FELT	3/8" FELT
1	3/8" FELT	3/8" FELT
1 1/2	3/8" FELT	3/8" FELT
2	1/2" FELT	WSW PADS
3	HN ISOLATORS	ND MOUNTS
4	HN ISOLATORS	ND MOUNTS
> 5	HS ISOLATORS 3/4" DEFL	V ISOLATORS 3/4" DEFL
* NOT REQUIRED FOR VENT STACKS, FIRE SPRINKLERS, OR GAS PIPING.		

help compensate for pipe misalignment and thermal expansion. Flexible pipe connections alone are usually not sufficient to isolate pipe transmitted vibrations but are part of an overall control strategy, which includes vibration isolation of the mechanical equipment and piping.

In high pressure hydraulic systems much of the vibration can be transmitted through the fluid so that pulse dampeners inserted in the pipe run can be helpful. These consist of a gas filled bladder, surrounding the fluid, into which the pressure pulse can expand and dissipate.

Where pipes are located in rated construction elements, closing off leaks at structural penetrations is critical to maintain the acoustical rating. Here the normal order of construction dictates the method of isolation. In concrete and steel structures, slabs are poured and then cored to accommodate pipe runs. In wood construction, piping is installed along with the framing, often preceding the pouring of any concrete fill. In both building types holes should be oversized by 1” (25 mm) more than the pipe diameter to insure that the pipe does not make direct structural contact. They are then stuffed with insulation, safing, or fire stop, and sealed. In slab construction the sealant can be a heavy mastic. With walls, the holes are covered with drywall leaving a 1/8” (3 mm) gap that is caulked. Pipe sleeves, which wrap the pipe at the penetration, are also commercially available. Details are shown in Fig. 11.17.

Electrical Connections

Where electrical connections are made to isolated equipment, the conduit must not short out the vibration isolation. If rigid conduit is used it should include a flexible section to isolate this path. The section should be long enough and slack enough that a 360° loop can be made in it.

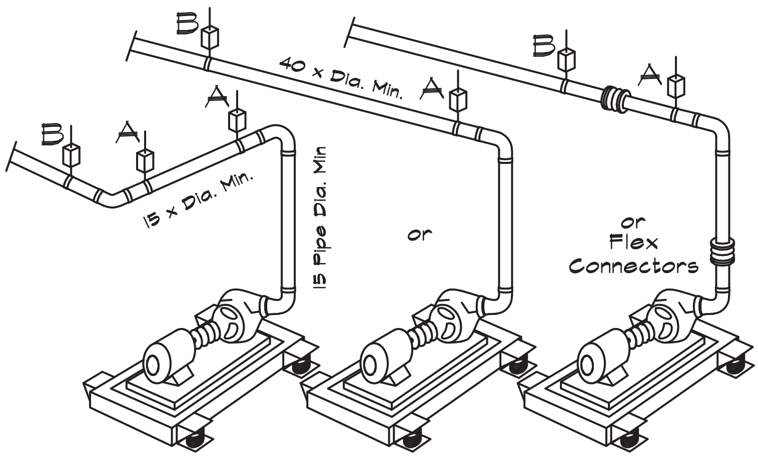
Duct Isolation

High-pressure ductwork having a static pressure of 4” (10 cm) or greater should be isolated for a distance of 30 ft (10 m) from the fan. Ducts are suspended on spring hangers with a minimum static deflection of 0.75” (19 mm), which should be spaced 10 ft (3 m) or less apart.

Roof-mounted sheet metal ductwork, located above sensitive occupancies such as studios, should be supported on vibration isolators having a deflection equal to that of the

FIGURE 11.16 Vibration Isolation of Piping and Ductwork (Vibron, 1989)

Vibration Isolation of Piping Connected to Isolated Equipment



Each of the above alternatives will result in equivalent piping isolation from a flexibly supported pump if the following suggestions are implemented.

(1) The pump inertia bases should be at least 2.5 times the unit weight.
(2) Piping isolators should have the following static deflections:
Points A and B - Equal to the pump base static deflection
Beyond Point B - At least 0.7" for 20 ft. from point B with isolators spaced at 10 ft. intervals. For critical locations the complete pipe run should be isolated, with isolator static deflection of at least 0.7" and spaced at 10 ft. intervals.
(3) For pipe diameters above 5", flexible connections are necessary.

Vibration Isolation of High Pressure Ductwork

High pressure (above 4" static pressure) duct runs should be isolated for a distance of 30 ft. from the fan. Ducts supported by hanger isolators should have a static deflection of at least 3/4" and be spaced 10 ft. apart.

FIGURE 11.17 Pipe or Duct Penetration

The opening is oversized to allow for the penetration and covered with the same number of layers as are on the wall.

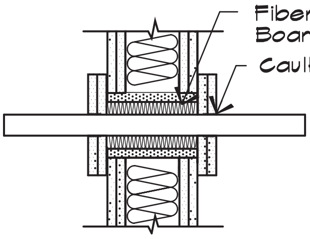
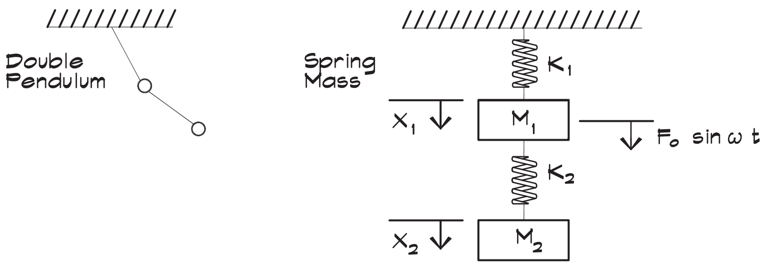


FIGURE 11.18 **Forced Excitation of an Undamped Two Degree of Freedom System (Ruzicka, 1971)**



isolated equipment to which they are attached, for the first three points of support. Beyond that point the ducts can be supported on mounts having half that deflection.

11.5 TWO DEGREE OF FREEDOM SYSTEMS

Two Undamped Oscillators

Although the one degree of freedom model is the most commonly utilized system for most vibration analysis problems, often situations arise that exhibit more complex motion. A model of a two degree of freedom system is shown in Fig. 11.18. This system consists of two masses and two springs with a sinusoidal force applied to one of the masses. The equations of motion can be written as

$$m_1 \ddot{x}_1 = k_2 (x_2 - x_1) - k_1 x_1 + F_0 \sin \omega t \tag{11.29}$$

$$m_2 \ddot{x}_2 = - k_2 (x_2 - x_1) \tag{11.30}$$

If we make the following substitutions

$$\omega_1 = \sqrt{k_1 / m_1} \qquad X_0 = F_0 / k_1$$

$$\omega_2 = \sqrt{k_2 / m_2}$$

and write the solution in terms of sinusoidal functions of displacement

$$x_1 = X_1 \sin \omega t$$

and

$$x_2 = X_2 \sin \omega t$$

Substituting these expressions into Eqs. 11.29 and 11.30, we obtain an expression for the relationship between the amplitude displacements

$$\left[1 + \frac{k_2}{k_1} - \left(\frac{\omega}{\omega_1} \right)^2 \right] X_1 - \left(\frac{k_2}{k_1} \right) X_2 = X_0 \tag{11.31}$$

and

$$-X_1 + \left[1 - \left(\frac{\omega}{\omega_2}\right)^2\right] X_2 = 0 \tag{11.32}$$

We can then study the system behavior by looking at the expressions for the ratio of the two amplitudes

$$\frac{X_1}{X_0} = \frac{\left[1 - (\omega/\omega_2)^2\right]}{\left[1 + \frac{k_2}{k_1} - \left(\frac{\omega}{\omega_1}\right)^2\right]\left[1 - \left(\frac{\omega}{\omega_2}\right)^2\right] - \frac{k_2}{k_1}} \tag{11.33}$$

$$\frac{X_2}{X_0} = \frac{1}{\left[1 + \frac{k_2}{k_1} - \left(\frac{\omega}{\omega_1}\right)^2\right]\left[1 - \left(\frac{\omega}{\omega_2}\right)^2\right] - \frac{k_2}{k_1}} \tag{11.34}$$

Now there are two resonant frequencies of the spring mass system, ω_1 and ω_2. From Eq. 11.33 we see that when the natural frequency of the second spring mass system matches the driving frequency of the impressed force, the numerator, and thus the amplitude X_1, goes to zero. At this frequency the amplitude of the second mass is

$$X_2 = -\frac{k_1}{k_2} X_0 = -\frac{F_0}{k_2} \tag{11.35}$$

where the minus sign indicates that the motion is out of phase with, and just counterbalances, the driving force. This is the principal behind a second form of vibration isolation known as mass absorption or mass damping. The absorber mass must be selected so as to match the applied force, taking into consideration the allowable spring deflection.

Two Damped Oscillators

Figure 11.19 gives the results of an imposed force on a damped two-degree of freedom spring mass system. The two resonant peaks are at different frequencies, with $\omega_2 > \omega_1$. In this example there is a relatively narrow frequency range where the second mass provides appreciable mass damping. Indeed it may generate an unwelcome resonant peak, slightly above the fundamental frequency of the second mass.

A mass absorber is most effective when it is used to damp the natural resonant frequency of the first spring mass system. If the ω_2 is selected to match ω_1, then the two resonant peaks coincide. When a broadband vibration or an impulsive load is applied to the system, the zero in the numerator in Eq. 11.33 smothers the resonant peaks and mass damping occurs. Figure 11.20 illustrates this case.

In long-span floor systems the floor itself acts like a spring mass system. A weight, suspended by isolator springs below a floor at a point of maximum amplitude, can be used as a dynamic absorber. These weights, which are usually 1% to 2% of the weight of the

FIGURE 11.19 Forced Response of a Two Degree of Freedom System (Ruzicka, 1971)

Transmissibility curves for a vibration isolation system to which is attached a passive vibration absorber tuned to a high frequency component of vibration excitation.

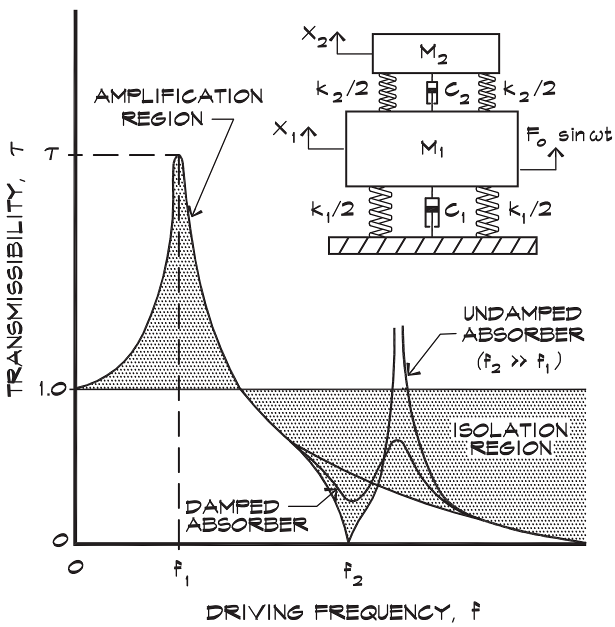
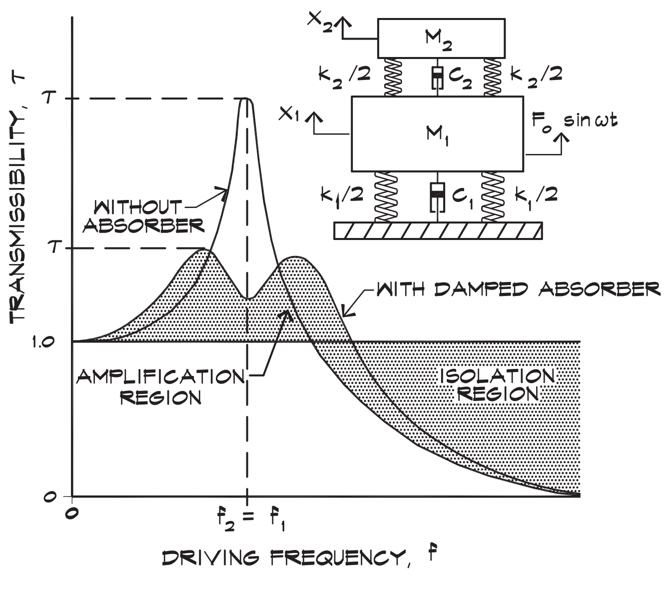


FIGURE 11.20 Forced Response of a Two Degree of Freedom System Near Resonance (Ruzicka, 1971)

Transmissibility curve for a spring mass system to which is attached a passive vibration absorber tuned so as to suppress the main system resonant response.



relevant floor area, are hung between the ceiling and the slab. It is not advisable to use the ceiling itself as the dynamic absorber, since mass damping works to minimize floor motion by maximizing the motion of the suspended mass. If the ceiling motion is maximized, it will radiate a high level of noise at the floor resonance.

Mass absorbers have also been used to damp the natural swaying motion of large towers such as the CN Tower in Toronto, Canada, using a dynamic pendulum. The double pendulum is another two degree of freedom system whose behavior is similar to that of a double spring mass. In this example the tower is encircled with a donut-shaped mass that is suspended as a pendulum. The mass is located at the point of maximum displacement of the normal modes of the structure. In the case of tall towers, the second and third modes are usually damped. The maximum displacement of the first mode occurs at the top of the tower and practical considerations prevent the suspension of a pendulum from this point. Two donut-shaped pendulums were used at the 1/3 and 1/2 points of the structure where they counter the second and third modes of vibration.

11.6 FLOOR VIBRATIONS

The vibration of floors due to motions induced by walking or mechanical equipment can be a source of complaints in modern building structures, particularly where lightweight construction such as concrete on steel deck, steel joists, or concrete on wood joist construction is used. Usually the vibration is a transient flexural motion of the floor system in response to impact loading from human activity (Allen and Swallow, 1975), which can be walking, jumping, or continuous mechanical excitation. The induced amplitudes are seldom enough to be of structural consequence; however, in extreme cases they may cause movement in light fixtures or other suspended items. The effects of floor vibrations are not limited to receivers located immediately below. With the advent of fitness centers, which feature aerobics, induced vibrations can be felt laterally 100 feet away on the same slab as well as up to 10 stories below (Allen, 1997).

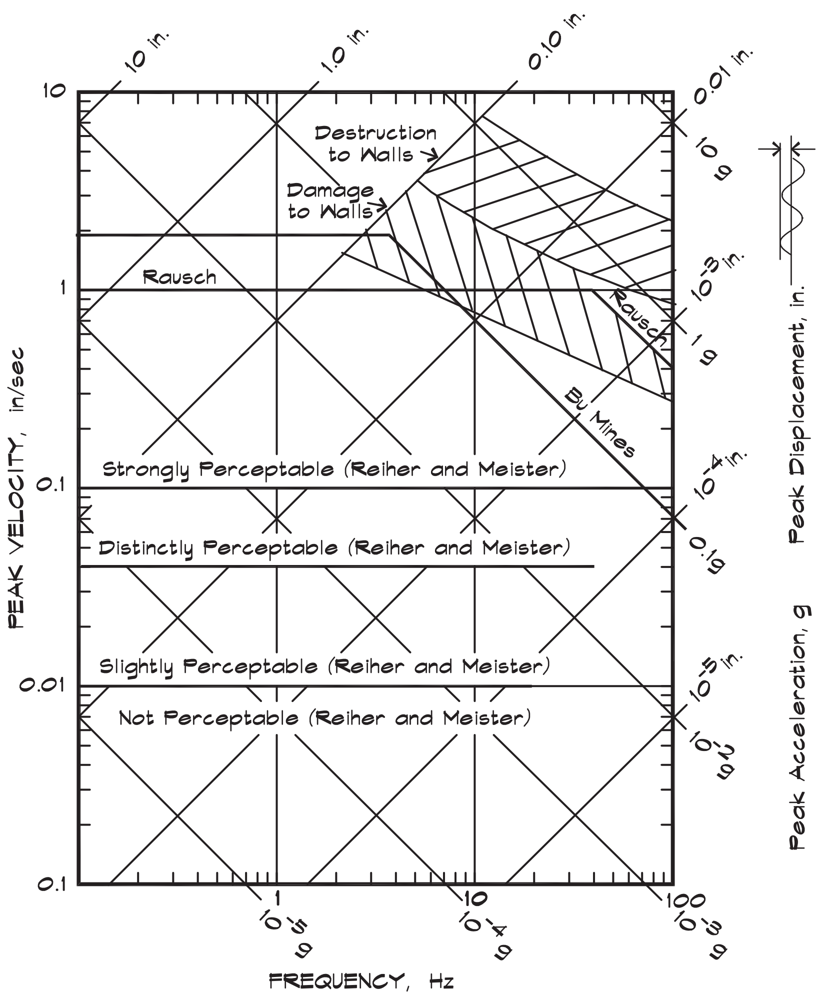
Sensitivity to Steady Floor Vibrations

People, equipment, and sophisticated manufacturing processes, such as computer chip production, are sensitive to floor vibrations. The degree of sensitivity varies with the process and various authors have published recommendations. One of the earliest was documented by Reiher and Meister (1931) and is shown in Fig. 11.21. These were human responses determined by standing subjects on a shaker table and subjecting them to continuous vertical motion. Subjects react more vigorously to higher velocities, and for high amplitudes, awareness increases with frequency. Also shown are the Rausch (1943) limits for machines and machine foundations and the US Bureau of Mines criteria for structural safety against damage from blasting.

Sensitivity to Transient Floor Vibrations

Vibrational excitation of floor systems may be steady or transient; however, it is usually the case that steady sources of vibration can be isolated. Transient vibrations due to footfall or other impulsive loads are absorbed principally by the damping of the floor. Damping provides a function somewhat akin to absorption in the control of reverberant sound in a room. People react, not only to the initial amplitude of the vibration, but also to its duration.

FIGURE 11.21 Response Spectra for Continuous Vibration (Richart et al., 1970; Reiher and Meister, 1931)



Investigators use tapping machines, walking at a normal pace (about 2 steps per second), and a heel drop test, where a subject raises up on his toes and drops his full weight back on his heels, as impulsive sources. This latter test represents a nearly worst-case scenario for human induced vibration, with aerobic studios and judo dojos being the exception.

After studying a number of steel-joist concrete-slab structures, Lenzen (1966) suggested that the original Reiher-Meister scale could be applied to floor systems having less than 5% of critical damping, if the amplitude scale were increased by a factor of 10. This means that we are less sensitive to floor vibration when it is sufficiently damped, in this case when only 20% of the initial amplitude remains after five cycles. He further suggested that if a vibration persists 12 cycles in reaching 20% of the initial amplitude, human response is the same as to steady vibration. Allen (1974), using his own experimental data along with observations of Goldman, suggested a series of annoyance thresholds for different levels of damping. This work, along with that of Allen and Rainer (1976), was adopted as a Canadian National Standard, which is shown in Fig. 11.22.

FIGURE 11.22 Annoyance Thresholds for Vibrations (Allen, 1974)

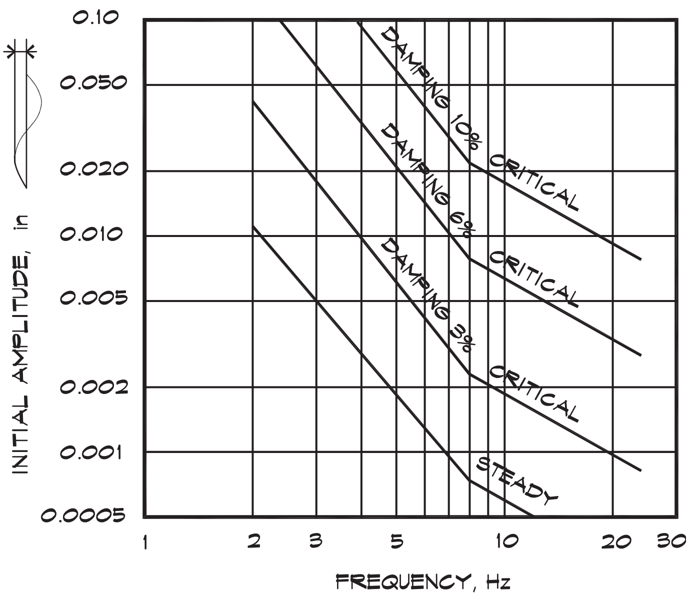
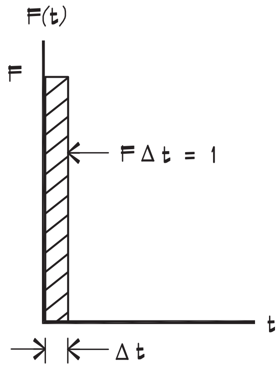


FIGURE 11.23 Impulsive Force



Vibrational Response to an Impulsive Force

When a linear system, such as a spring mass damper, is driven by an impulsive force we can calculate the overall response. For the study of vibrations in buildings the system of interest here is a floor and the impulsive force is a footfall generated by someone walking. An impulse force is one in which the force acts over a very short period of time. An impulse can be defined as

$$\hat{F} = \int_{t}^{t+\Delta t} F \, dt \cong F \, \Delta t \tag{11.36}$$

Figure 11.23 shows an example of an impulsive force, having a magnitude F and a duration Δt. An impulsive force, such as a hammer blow, can be very large; however, since it occurs over a rather short period of time, the impulse is finite. When the impulse is normalized to 1 it is called a unit impulse.

FIGURE 11.24 Response of a Damped System to a Delta Function Impulse $\widehat{F}$ (Thomson, 1965)

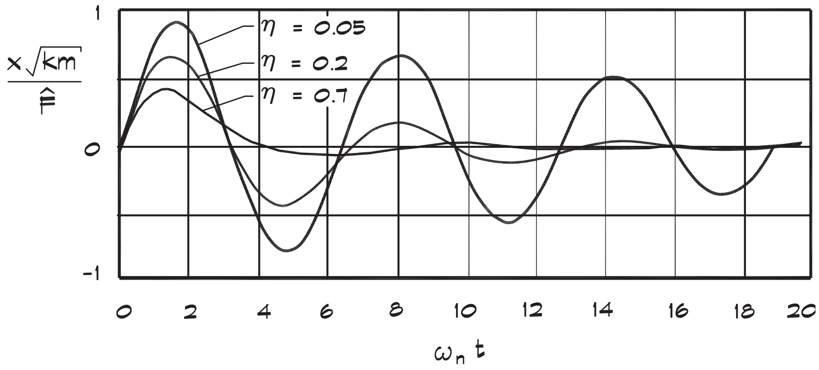


Figure 11.24 illustrates the response of a damped spring mass system under an impulse force for various values of the damping coefficient. From Newton's law, $F\,\Delta t = m\,\dot{x}_2 - m\,\dot{x}_1$. When an impulsive force is applied to a mass for a short time the response is a change in velocity without an appreciable change in displacement. The velocity changes rapidly from zero to an initial value of $\hat{F}/m$. We can use this as the initial boundary condition, assuming an initial displacement of zero, by plugging into the general undamped solution (Eq. 11.6). We get the response to the impulse force

$$x = \frac{\hat{F}}{m\,\omega_n} \sin \omega_n t \tag{11.37}$$

where ω_n is the undamped natural frequency of the spring mass system. If the system is damped, we can use the same procedure to calculate the response by plugging into Eq. 11.12.

$$x = \frac{\hat{F}}{m\,\omega_n\,\sqrt{1 - \eta^2}}\, e^{-\,\eta\,\omega_n\,t} \quad \sin\left(\sqrt{1 - \eta^2}\;\omega_n\, t\right) \tag{11.38}$$

Response to an Arbitrary Force

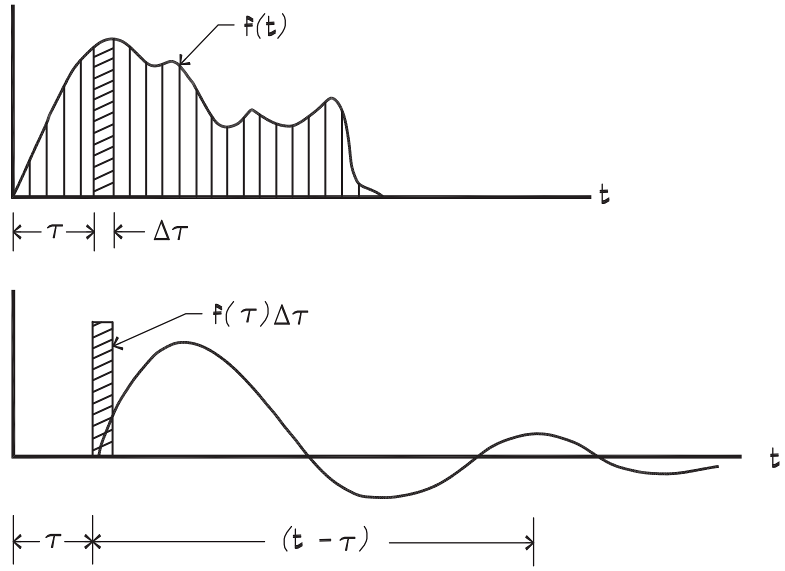
The impulse response in Eq. 11.38 is a fundamental property of the system. It is given a special designation, g (t), where $x = \hat{F}\, g\,(t)$. Once the system response to a unit impulse (sometimes called a delta function) has been determined, it is possible to calculate the response to an arbitrary force f (t) by integrating (summing) the effects of a series of impulses as illustrated in Fig. 11.25.

At a particular time τ, the force function has a value, which can be described by an impulse $\hat{F} = f(\tau)\ \Delta\ \tau$. The contribution of this slice of the force function on the system response at some elapsed time $t - \tau$ after the beginning of that particular pulse is given by

$$x = f(\tau)\ \Delta\tau\ \ g\,(t - \tau) \tag{11.39}$$

and the response to all the small force pulses is given by integrating over the total time, t_p, the force is applied. If the time of interest is less than t_p, the limit of integration becomes the

FIGURE 11.25 An Arbitrary Pulse as a Series of Impulses (Thomson, 1965)



time of interest.

$$x\,(t) = \int_0^{t_p} f(\tau)\; g\,(t - \tau)\; d\,\tau \tag{11.40}$$

This integral is known by various names including the Duhamel integral, the summation integral, and the convolution integral. It says that if we know the system impulse response, we can obtain the system response for any other type of input by performing the integration. This has profound implications for the modeling of concert halls and other spaces since the impulse response of a room can be modeled and the driving force can be music. Thus we can listen to the sound of a concert hall before it is built.

Response to a Step Function

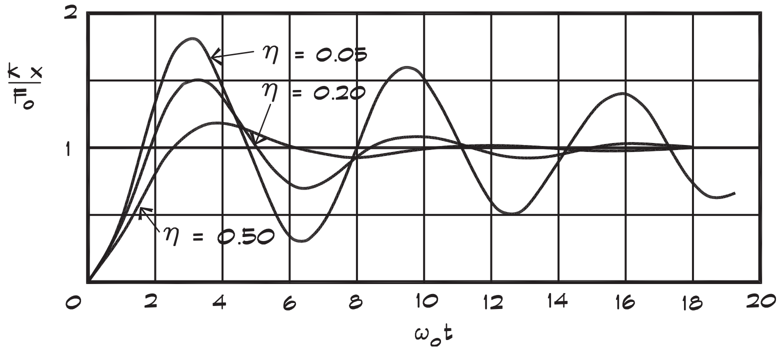
If the shape of a force applied to a spring mass system consists of a constant force that is instantaneously applied, we can substitute the force time behavior, $f\,(t) = F_0$, into Eq. 11.40 along with the system response to obtain the response behavior. For an undamped spring mass system the result is

$$x\,(t) = \int_0^{t} \frac{F_0}{m\,\omega_n} \sin\omega_n\;(t - \tau)\; d\,\tau \tag{11.41}$$

which is

$$x\,(t) = \frac{F_0}{k}\,(\,1 - \cos\omega_n\; t) \tag{11.42}$$

FIGURE 11.26 Response of a Damped System to a Unit Step Function (Thomson, 1965)



and for the damped system the result is (see Harris and Crede, 1961; or Thomson, 1965)

$$x = \frac{F_0}{k}\left[1 - \frac{e^{-\eta\,\omega_n\,t}}{\sqrt{1-\eta^2}}\cos\left(\sqrt{1-\eta^2}\,\omega_n\,t - \psi\right)\right] \tag{11.43}$$

where $\tan\psi = \dfrac{\eta}{\sqrt{1-\eta^2}}$

Figure 11.26 shows the system response for a damped spring mass as a function of damping. When the damping is zero the maximum amplitude is twice the displacement that the system would experience if the load were applied slowly.

Vibrational Response of a Floor to Footfall

A footstep consists of two step functions, one when the load is applied and one when it is released. Ungar and White (1979) have modeled this behavior using a versed sine pulse in Fig. 11.27, and have calculated the envelope for the dynamic amplification, defined as the ratio of the maximum dynamic amplitude divided by the static deflection obtained under the load, F_m.

$$A_m = \frac{X_{max}}{X_{static}} = \frac{\sqrt{2\left(1 + \cos 2\pi\ f_n\ t_0\right)}}{\left[1 - \left(2\ f_n\ t_0\right)^2\right]} \tag{11.44}$$

FIGURE 11.27 Idealized Footstep Force Pulse (Ungar and White, 1979)

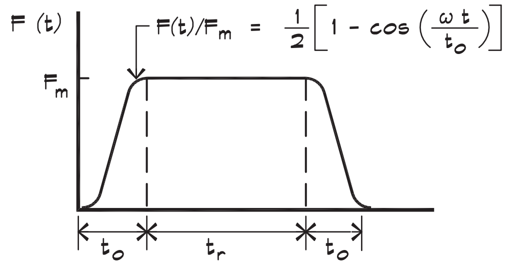
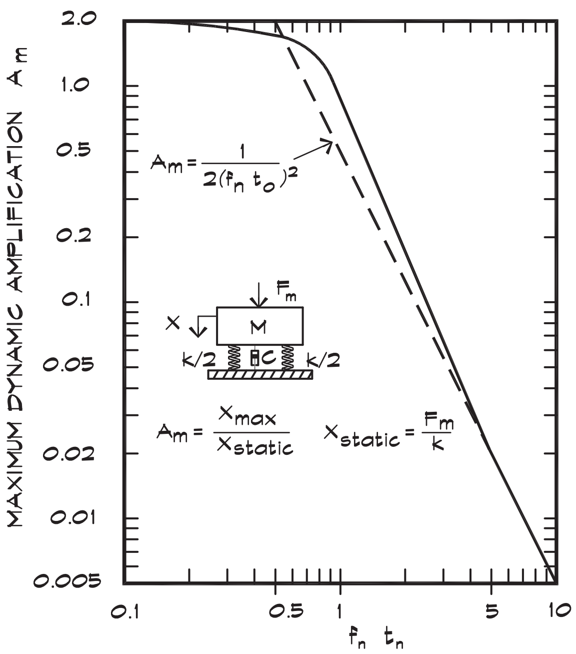


FIGURE 11.28 Maximum Dynamic Deflection Due to a Footstep Pulse (Ungar and White, 1979)



where $f_n = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$ and t_0 is the rise time of the pulse. Note that k is the stiffness at the point where the footstep is taken. This equation does not give us the detailed behavior of the motion but gives us the envelope of the maximum deflection with resonant frequency, which is often sufficient for design purposes. For values of $f_n\ t_0$ that are small when compared to 1, the maximum dynamic amplification $A_m \cong 2$. For large values of $f_n\ t_0$, the amplification becomes $A_m \cong a / (2\ f_n\ t_0)^2$, where a varies between 0 and 2, so that under these conditions $A_m \le 1/\left[2\left(f_n\ t_0\right)^2\right]$. Figure 11.28 gives a plot of the upper bound envelope for A_m. In Eq. 11.44 we note that the product $f_n\ t_0$ is equal to t_0 / t_n, the ratio of the pulse rise time to the natural period of floor vibration.

Figure 11.29 shows published data on footstep forces generated by a 150 lb (68 kg) male walker, and Fig. 11.30 shows the dependence of the rise time and force on walking speed. The figures allow us to estimate the maximum deflection of a floor system for various values of the resonant floor frequency.

While floors have a multitude of vibrational modes, the fundamental is usually the most important. It exhibits the lowest resonant frequency, is the most directly excitable structural motion, and has the softest (lowest impedance) point at its antinode. Some measured results are shown in Fig. 11.31 for a concrete I-beam structure. Although only two floor modes have been predicted, and floors are not pure undamped spring mass systems, the curve neatly bounds the remainder of the modes.

Control of Floor Vibrations

When it is desirable to control floor vibration for human comfort, it is important to limit the maximum amplitude as well as increase the damping. If the driving force is footfall, we

FIGURE 11.29 The Footstep Force Pulse Produced by a 150 lb (68 kg) Male Walker (Ungar and White, 1979)

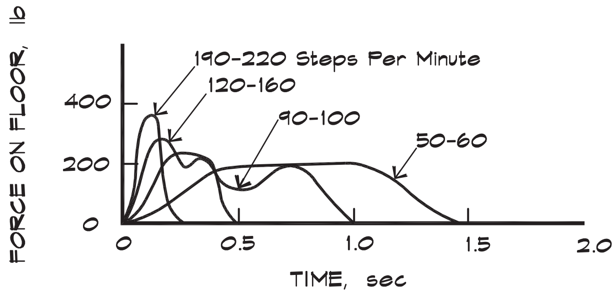
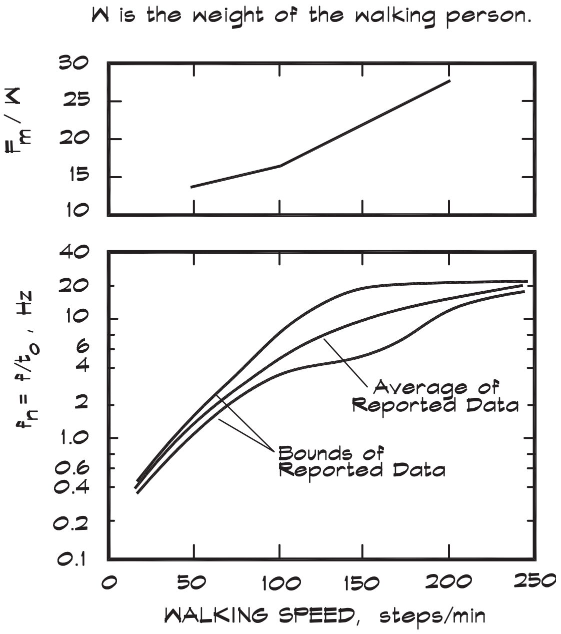


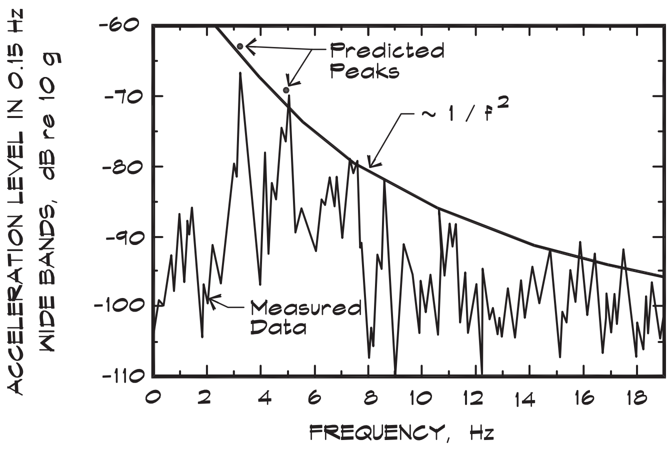
FIGURE 11.30 Dependence of the Maximum Force F and the Rise Time t of a Footstep Pulse on the Walking Speed (Ungar and White, 1979)



can use the amplification factor rise time t_0 to the natural period t_n of the structural mode. When the pulse rise time is a small fraction of the natural period we might expect a different behavior than for cases where the rise time is a large multiple of the period. This is illustrated in Fig. 11.28. From the graph it is reasonable to take the value of $f_n\ t_0 = 0.5$ as the dividing point between these two regions. From Fig. 11.29, the rise time for a typical rapid walker is about a tenth of a second, which means that the dividing point corresponds to a floor resonance of about 5 Hz. The fundamental resonances of most concrete floor systems fall into the region between 5 and 8 Hz, so that rapid walking on these structures corresponds to the region where $f_n\ t_0 \geq 0.5$. For this region,

$$x_{max} = F_m / 2\,k\,(f_n\ t_0)^2 \cong 2\,\pi^2\,F_m\,M / t_0^{\ 2}\,k^2 \qquad (11.45)$$

FIGURE 11.31 Footfall Response of a Concrete I-Beam Floor Structure (Ungar and White, 1979)



and

$$a_{max} \cong (2\,\pi\,f_n)^2\ x_{max} = 2\,\pi^2\,F_m\,/ = t_0{}^2\,k \tag{11.46}$$

where a_{max} represents the maximum floor acceleration, k the local modal stiffness, and M the corresponding mass. It is clear that the structural stiffness is the most important component in decreasing both the maximum amplitude and the maximum acceleration. The floor mass does not appear in the equation for acceleration. The maximum displacement increases with mass, unless the mass increases the stiffness.

In the region where $f_n\ t_0 \leq 0.5$, which would correspond to a very long span floor, we find that

$$x_{max} \cong F_m\,/\,k \tag{11.47}$$

and

$$a_{max} \cong (2\,\pi\,f_n)^2\ x_{max} = 2\,F_m\,/\,M \tag{11.48}$$

Here only the stiffness affects the maximum displacement and only the mass affects the maximum acceleration.

Allen and Swallow (1974) have addressed the design of concrete floors for vibration control. It is difficult to change the fundamental resonant frequency. A concrete floor might weigh 200,000 lbs (91,000 kg) and changing the gross physical properties requires major structural changes. Damping, however, is a factor that produces significant results and may be easier to control. These authors make the following preconstruction design considerations:

1. Cross bracing in steel structures has little effect (Moderow, 1970).
2. Noncomposite construction tends to increase damping by 1 to 2% over composite construction (Moderow, 1970).
3. Concrete added to the lower cord of the structural steel can increase damping of a completed floor by 2%.

4. Increasing the thickness of the concrete slab decreases the maximum amplitude and the natural frequency and increases the damping.
5. Cover plates on the joists increase the natural frequency and decrease amplitudes, due to the increased stiffness of the floor. When the data are plotted to determine human response it is found that the change moves downward with frequency, essentially paralleling the human response curve, so little is gained.

After construction, there are still some therapeutic measures available, principally to increase damping. Partitions are very effective in adding damping to an existing structure and can increase the overall damping to 14% of critical. Even lightweight low partitions, planter boxes, and the like can increase damping to 10% of critical. Partitions may be attached to a slab either above or below. Damping posts at critical locations can improve damping somewhat, but they may interfere with the decor. A dynamic absorber can be hung from a floor and can include a damper as part of the design. Allen and Swallow (1975) report that a mass damper tuned to 0.9 of the fundamental frequency and with 10% of critical damping reduced the floor amplitude by 50% and increased floor damping from 3 to 15% of critical. The added mass was 1 percent of the total floor mass.