6 WAVE ACOUSTICS

Much of architectural acoustics can be addressed without consideration of the wave nature of sound. For example, environmental acoustics and the transmission of outdoor sound, for the most part, can be visualized and modeled as a flow of energy from point to point, although many effects, such as ground and barrier attenuation, are frequency dependent. Nevertheless, for many critical aspects of acoustics, knowledge of wave phenomena is essential. Wave acoustics takes into account fundamental properties that are wavelength and phase dependent, including the scaling of interactions to wavelength, the phenomenon of resonance, and the combination of amplitudes based not only on energy but also on phase.

6.1 RESONANCE

Simple Oscillators

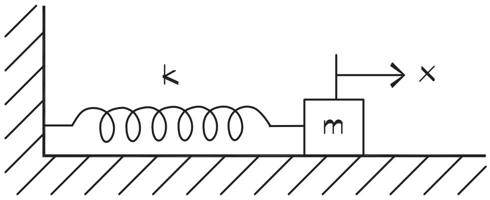
Many mechanical systems have forces that restore a body to its equilibrium position after it has been displaced. Examples include a spring mass, a child's swing, a plucked string, and the floor of a building. When such a system is pulled away from its rest position, it will move back toward equilibrium, transition through it, and go beyond only to return again and repeat the process. All linear oscillators are constrained such that, once displaced, they return to the initial position. The movement repeats at regular intervals, which have a characteristic duration and thus a characteristic frequency, called the natural frequency or resonant frequency of the system.

Although many such systems exist, the simplest mechanical model is the spring and mass shown in Fig. 6.1. A frictionless mass, m, is attached to a linear spring, whose restoring force is proportional to the displacement away from equilibrium, x. This relationship is known as Hooke's law and is usually written as

$$\mathrm{F} = - k \, \mathrm{x} \tag{6.1}$$

where k is the constant of proportionality or the spring constant.

FIGURE 6.1 A Simple Spring-Mass System



When the body is in motion, inertial forces, due to the mass, counterbalance the spring force, according to Newton's second law

$$F = m\,a = m\,\frac{d^2x}{d\,t^2} \tag{6.2}$$

where F = force applied to the mass, (N)

m = mass, (kg)

a = acceleration or the second time derivative of the displacement, (m/s^2)

The forces in a simple spring mass system are: 1) the spring force, which depends on the displacement away from the equilibrium position, and 2) the inertial force of the accelerating mass. The equation of motion is simply a summation of the forces on the body

$$m\,\frac{d^2\,x}{d\,t^2} + k\,x = 0 \tag{6.3}$$

If we introduce the quantity

$$\omega_n^2 = \frac{k}{m} \tag{6.4}$$

the equation can be written as

$$\frac{d^2\,x}{d\,t^2} + \omega_n^2\,x = 0 \tag{6.5}$$

The solution requires that the second derivative be the negative of itself times a constant. This is a property of the sine and cosine functions, so we can write a general solution as

$$x = A\,\sin\,\omega_n\,t + B\,\cos\,\omega_n\,t \tag{6.6}$$

where A and B are arbitrary constants defined by the initial conditions. If the system is started at $t = 0$ from a position x_0 and a velocity v_0, then the constants are defined and the equation becomes

$$x = \frac{v_0}{\omega_n}\,\sin\,\omega_n\,t + x_0\,\cos\,\omega_n\,t \tag{6.7}$$

Since we can write the coefficients in Eq. 6.6 in terms of trigonometric functions

$$a\,\sin\,\omega\,t + b\,\cos\,\omega\,t = \cos\,\phi\,\cos\,\omega\,t - \sin\,\phi\,\sin\,\omega\,t = \cos(\omega\,t + \phi) \tag{6.8}$$

and we can form a second general solution

$$x = X \cos(\omega_n t + \phi) \tag{6.9}$$

where

$$X = \sqrt{x_0^2 + (v_0 / \omega_n)^2} \quad \text{and} \quad \phi = \tan^{-1}\left[\frac{-v_0}{\omega_n x_0}\right] \tag{6.10}$$

and X = amplitude of the maximum displacement of the system, (m)
ϕ = initial phase of the system, (rad)

According to Eq. 6.9 the behavior of the spring mass system is harmonic, with the motion repeating every time period $t = T$, where $\omega_n T = 2\pi$. The period is

$$T = 2\pi\sqrt{m / k} \tag{6.11}$$

and the natural frequency of vibration is the reciprocal of the period

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{6.12}$$

The natural frequency of an undamped spring mass system can be specified in terms of the deflection δ of the spring, under the load of the mass since $k\,\delta$ = force = m g, where g is the acceleration due to gravity. Using $g = 386\ \text{in/s}^2 = 9.8\ \text{m/s}^2$, the natural frequency can be written as

$$f_n = \frac{3.13}{\sqrt{\delta_i}} = \frac{5}{\sqrt{\delta_{cm}}} \tag{6.13}$$

where f_n = natural frequency of the system, (Hz)
δ = deflection of the spring under the weight of the mass – δ_i in inches and δ_{cm} in centimeters

This provides a convenient way of remembering the natural frequency of a spring mass system. A one-inch deflection spring is a 3-Hz oscillator, and a one-centimeter deflection spring is a 5-Hz oscillator. These simple oscillators appear over and over in various forms throughout architectural acoustics.

Air Spring Oscillators

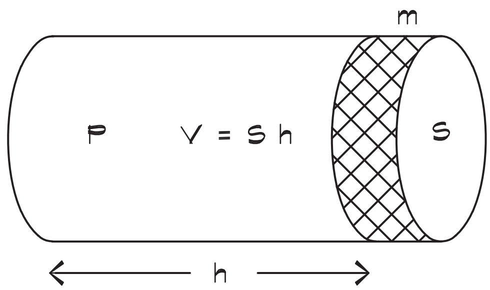
When air is contained in an enclosed space, it can act as the spring in a spring mass system. In the example shown in Fig. 6.2, a frictionless mass is backed by a volume of air. When the mass moves into the volume, the pressure increases, creating a force that opposes the motion. The spring constant of the enclosed air is derived from the equation of state

$$P\,V^{\gamma} = \text{constant} \tag{6.14}$$

differentiating

$$\gamma P\,V^{\gamma-1}\,dV + V^{\gamma}\,dP = 0 \tag{6.15}$$

FIGURE 6.2 A Frictionless Mass and an Air Spring



The rate of change of pressure with volume is

$$dP = -\frac{\gamma P}{V}\, dV \tag{6.16}$$

from which we can obtain the spring constant for a trapped air volume of depth h

$$k = \frac{\gamma\ P\ S}{V}\,\frac{dV}{dh} = \frac{\gamma\ P\ S^2}{V} = \frac{\gamma\ P\ S}{h} \tag{6.17}$$

and the natural frequency is

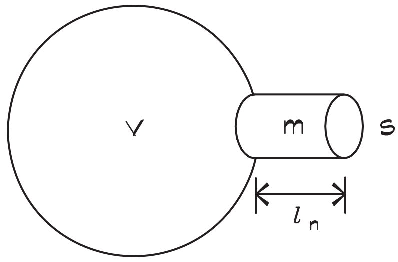
$$f_n = \frac{1}{2\,\pi}\,\sqrt{\frac{\gamma\ P_0\ S}{m\ h}} \tag{6.18}$$

where f_n = resonant frequency of the system, (Hz)
P_0 = atmospheric pressure, (1.013×10^5 Pa)
m = mass of the piston, (kg)
γ = ratio of specific heats = 1.4 for air
S = area of the piston (m^2)
h = depth of the air cavity (m)

For normal atmospheric pressure, the resonant frequency of an unsupported panel of drywall, 16 mm thick (5/8") weighing 12.2 kg/sq m (2.5 lbs/sq ft) with a 9 cm (3.5") air cavity behind it, is about 18 Hz. If the air gap is reduced to 1.2 cm (about 1/2 "), the natural frequency rises to about 50 Hz. If the drywall panel were supported on resilient channel having a deflection due to the weight of the drywall of 1 mm, its spring mass resonant frequency would be about 16 Hz. The presence of the air spring caused by the 1/2 " air gap stiffens the connection between the drywall and the support system, which results in poor vibration isolation when resilient channel is applied directly over another panel. A similar condition occurs in the construction of resiliently mounted (floating) floors if the trapped air layer is very shallow.

The air spring was the basis behind the air-suspension loudspeaker system, which was developed in the 1960s. The idea was that a cone loudspeaker could be made lighter and more compliant (less stiff) if the spring constant of the air in the box containing it made up for the lack of suspension stiffness. The lighter cone made the loudspeaker easier to accelerate and thus improved its high-frequency response.

FIGURE 6.3 Geometry of a Helmholtz Resonator



Helmholtz Resonators

A special type of air-spring oscillator is an enclosed volume having a small neck and an opening at one end. It is called a *Helmholtz resonator*, named in honor of the man who first calculated its resonant frequency. Referring to the dimensions in Fig. 6.3, the system mass is the mass of the air in the neck, $m = \rho_0 \, S \, l_n$ and the spring constant k is $\gamma \, P_0 \, S^2 / V$. Using the relationship shown in Eq. 6.12 the natural frequency is given by

$$f_n = \frac{c_0}{2\,\pi} \sqrt{\frac{S}{V\, l_n}} \tag{6.19}$$

Note that the frequency increases with neck area and decreases with enclosed volume and neck length.

Helmholtz resonators are used in bass-reflex or ported loudspeaker cabinets to extend the bass response of loudspeakers by tuning the box so that the port emits energy at low frequencies. The box-cone combination is not a simple Helmholtz resonator, but acts like a high-pass filter. Thiele (1971) and Small (1973) did extensive work on developing equivalent circuit models of the combined system, which is designed so that its resonant frequency is just below the point at which the loudspeaker starts to loose efficiency. An example of a low-frequency loudspeaker response curve is shown in Fig. 6.4. Ported boxes radiate low-frequency sound out the opening, where it combines in phase with the direct cone radiation. Potentially detrimental high-frequency modes, due to resonances within the cavity, are dampened by filling the box with fiberglass.

A similar technique is used to create tuned absorbers in studios and control rooms, where they are called bass traps. Relatively large volumes, filled with absorption, are tucked into ceilings, under platforms, and behind walls to absorb low-frequency sound. To act as a true Helmholtz resonator they must have a volume, a neck, and an opening, whose dimensions are small compared with the wavelength of sound to be absorbed.

Neckless Helmholtz Resonators

When an enclosed volume has an opening in it, it can still oscillate as a Helmholtz resonator even though it does not have an obvious neck. Since the air constricts as it passes through the opening, it forms a virtual tube, illustrated in Fig. 6.5, sometimes called a *vena contrata* in fluid dynamics. This acts like a neck, with an effective length equal to the thickness of the plate l_0 plus an additional "end effect" factor of (0.85 a), where a is the radius of the opening.

FIGURE 6.4 Frequency Response of a Sealed Cabinet vs a Bass-Reflex Cabinet (Roozen et al., 1998)

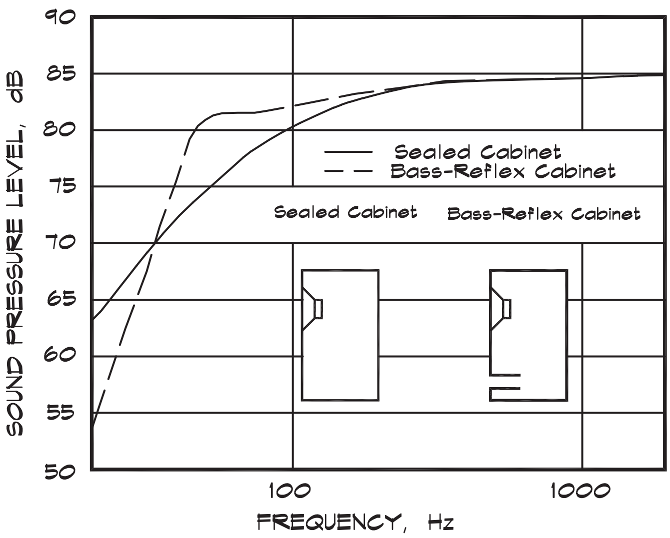
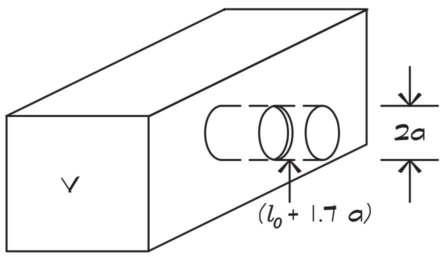


FIGURE 6.5 Geometry of a Neckless Helmholtz Resonator



Since there are two open ends of the tube, the natural frequency becomes

$$f_n = \frac{c_0}{2\,\pi} \sqrt{\frac{\pi\, a^2}{V\,(l_0 + 1.7\,a)}} \tag{6.20}$$

When there are multiple openings in the side of an enclosure, such as with a perforated plate spaced out from a wall, a Helmholtz resonance effect can still occur. The length of the neck is still the same as that used in Eq. 6.20, but the open area is the area of each hole times the number of holes in the plate.

For a perforated panel located a distance d from a solid wall, the resonant frequency is

$$f_n = \frac{c_0}{2\,\pi} \sqrt{\frac{\sigma}{d\,(l_0 + 1.7\,a)}} \tag{6.21}$$

where σ = fraction of open area in the panel which for round holes, staggered is 0.9 $(2a/b)^2$ and for round holes, straight is 0.785 $(2a/b)^2$, where b is the hole spacing

d = depth of the airspace behind the panel in units consistent with those of l_0, a, and c_0

6.2 WAVE EQUATION

One-Dimensional Wave Equation

The wave equation is a differential equation that formally defines the behavior in space and time of the pressure, density, and other variables in a sound wave. It is rarely used directly in architectural acoustics, but its solutions are the basis of wave acoustics, which is important to the understanding of many phenomena. Its derivation includes several assumptions about the nature of the medium through which sound passes. It assumes that the conducting medium follows the equation of continuity (conservation of mass), Newton's second law of motion, and an equation of state, relating the pressure and density. Refer to Kinsler et al., (1982) for a more detailed treatment.

To derive the wave equation we examine a small slice of a fluid (such as air), having thickness dx, shown from one side in Fig. 6.6. As a sound wave passes by, the original dimensions of the box ABCD move, in one dimension, to some new position A'B'C'D'. If S is the area of the slice, having its normal along the x axis, then there is a new box volume

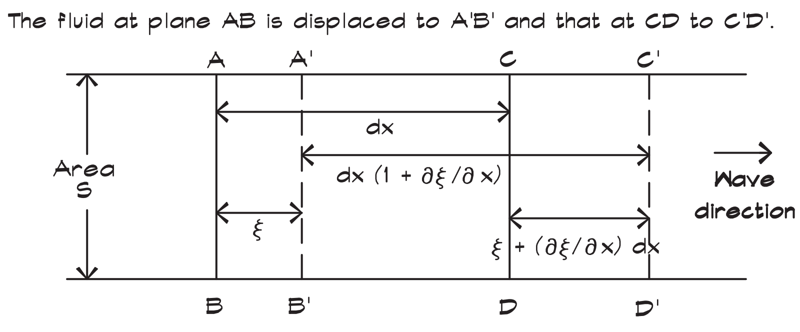
$$V + dV = S\ dx \left(1 + \frac{\partial \xi}{\partial x}\right) \tag{6.22}$$

where ξ is the displacement of the slice. In Chapt. 2, the bulk modulus was defined in Eq. 2.39 as the ratio of the change in pressure to the fractional change in volume

$$dP = -\ B\ \frac{d\ V}{V} \tag{6.23}$$

In terms of the volume in Fig. 6.6, the change in the total pressure P is the acoustic pressure p and the change in volume is S dx. So comparing Eqs. 6.22 and 6.23, we obtain the relation

FIGURE 6.6 The Fluid Displacement during the Passage of a Plane Wave (Rossing and Fletcher, 1995)



between the acoustic pressure and the change in length. The minus sign means that if the length decreases, the pressure increases.

$$p = -\,B\,\frac{\partial \xi}{\partial x} \tag{6.24}$$

The motion of the slice is described by Newton's second law, so we set the pressure gradient force equal to the slice mass times its acceleration,

$$-\,S\left(\frac{\partial p}{\partial x}\,d\,x\right) = \rho_0\,S\,dx\,\frac{\partial^2\,\xi}{\partial\,t^2} \tag{6.25}$$

and simplifying we obtain an expression known as Euler's equation

$$-\,\frac{\partial p}{\partial x} = \rho_0\,\frac{\partial^2 \xi}{\partial\,t^2} \tag{6.26}$$

Then using Eqs. 6.24 and 6.26

$$\frac{\partial^2 \xi}{\partial\,t^2} = \frac{B}{\rho_0}\,\frac{\partial^2 \xi}{\partial\,x^2} \tag{6.27}$$

By differentiating Eq. 6.26 once with respect to x, and Eq. 6.24 twice with respect to t, and adding them

$$\frac{\partial^2 p}{\partial\,t^2} = c^2\,\frac{\partial^2 p}{\partial\,x^2} \tag{6.28}$$

where we have used Eq. 2.40 for the speed of sound. This is the one-dimensional wave equation, expressed in terms of pressure. It relates the spatial and time dependence of the sound pressure within the wave.

Solutions to Eq. 6.28 are not difficult to find; in fact, any pressure wave that is a function of the quantity $(x \pm c\,t)$ will do. This reveals the infinite number of possible waveforms that a sound wave can take. The plus and minus signs indicate the direction of propagation of the wave. Although many functions are solutions to the wave equation, not all are periodic. In architectural acoustics, the periodic solutions are of primary interest to us, although nonperiodic phenomena such as sonic booms are occasionally encountered. A periodic solution, which describes a plane wave traveling in the $+\,x$ direction is

$$\mathbf{p} = \mathbf{A}\,e^{-j\,k\,x}\,e^{j\,\omega\,t} = \mathbf{A}\,\cos\left(-k\,x + \omega\,t\right) \tag{6.29}$$

where $\mathbf{p}$ = acoustic pressure, (Pa)
$\mathbf{A}$ = maximum pressure amplitude, (Pa)
$j = \sqrt{-1}$
ω = radial frequency, (rad / s)
k = wave number = ω / c, (rad / m)

The right side of Eq. 6.29 is a familiar form, which we derived using heuristic arguments in Eq. 2.32. The terms on the left are another way of expressing periodic motion using exponentials, which are more convenient for many calculations.

Using Eq. 6.26, the particle velocity can be obtained

$$-\frac{\partial p}{\partial x} = \rho_0 \frac{\partial u}{\partial t} \tag{6.30}$$

Assuming that u has the same form as Eq. 6.29, the time derivative can be replaced by j ω or j c k

$$\mathbf{u} = \frac{j}{k\,\rho_0\,c} \frac{\partial \mathbf{p}}{\partial x} \tag{6.31}$$

Three-Dimensional Wave Equation

When dealing with sound waves in three dimensions we have a choice of several coordinate systems. Depending on the nature of the problem, one system may be more appropriate than another. In a rectilinear (x, y, z) system, we can write separate equations in each direction similar to Eq. 6.28 and combine them to obtain

$$\frac{\partial^2 p}{\partial t^2} = c^2 \left[\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2}\right] = c^2\,\nabla^2 p \tag{6.32}$$

The term in the bracket in Eq. 6.32 is called the Laplace operator and is given the symbol ∇^2.

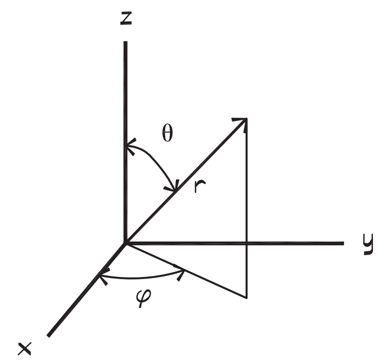
In the spherical coordinate system in Fig. 6.7, there are two angular coordinates, usually designated θ and ϕ, and one radial coordinate designated r. The Laplace operator in spherical coordinates is

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right) + \frac{1}{r^2\,\sin\,\theta}\frac{\partial}{\partial\,\theta}\left(\sin\,\theta \frac{\partial}{\partial\,\theta}\right) + \frac{1}{r^2\,\sin^2\,\phi}\frac{\partial^2}{\partial\,\phi^2} \tag{6.33}$$

For a nondirectional source we can dispense with consideration of the angular coordinates and examine only the dependence on r. This yields the one-dimensional wave equation for a spherical wave

$$\frac{\partial^2 p}{\partial t^2} = c^2 \left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\,\frac{\partial p}{\partial\,r}\right)\right] \tag{6.34}$$

FIGURE 6.7 The Spherical Coordinate System



6.3 SIMPLE SOURCES

Monopole Sources

The general solution to Eq. 6.34 for a wave moving in the positive r direction is

$$\mathbf{p} = \frac{\mathbf{A}}{r}\, e^{-j k r}\, e^{j \omega t} \tag{6.35}$$

Using Eq. 6.31, we can solve for the particle velocity

$$\mathbf{u} = \frac{\mathbf{A}}{r\, \rho_0\, c_0} \left(1 + \frac{1}{j k r}\right) e^{-j k r}\, e^{j \omega t} \tag{6.36}$$

When we are far away from the source and k r >> 1, the particle velocity reverts to its plane wave value, $\mathbf{p} / \rho_0\, c_0$. Equations 6.35 and 6.36 describe the behavior of a simple point source, sometimes called a monopole.

Doublet Sources

When two point sources are placed at x = 0 and x = d, they form an acoustic doublet. The sources may be in phase or out of phase. For this analysis, they are assumed to be radiating at the same source strength and frequency. Figure 6.8 shows the geometry for this configuration. When the receiver is close to the source the geometrical relationships are a bit complicated. If the receiver is far away, we can make the approximation that the lines between the sources and the receiver are almost parallel. Under these constraints, the pressure for the two sources combined is

$$\mathbf{p} = \frac{\mathbf{A}}{r}\, e^{j \omega t}\, e^{-j k r} \left(1 \pm e^{j k d \sin \theta}\right) \tag{6.37}$$

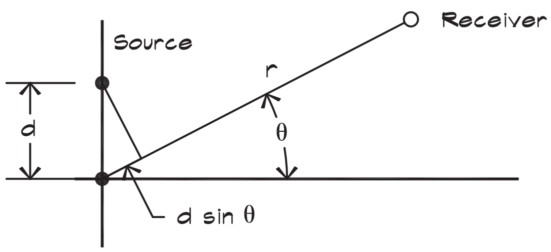
where the plus sign is for in-phase sources and the minus sign is for out-of-phase sources. The value of the pressure for in-phase sources is

$$p = \frac{2\, A}{r} \cos\left(\frac{1}{2}\, k\, d\, \sin\, \theta\right) \tag{6.38}$$

and for out-of-phase sources is

$$p = \frac{2\, A}{r} \sin\left(\frac{1}{2}\, k\, d\, \sin\, \theta\right) \tag{6.39}$$

FIGURE 6.8 The Geometry of a Doublet Source



The total power radiated by the doublet can be calculated by integrating the square of the pressure over all angles

$$W = \frac{1}{2} \iint \left(\frac{p^2}{\rho_0 \, c_0} \right) r^2 \sin \theta \, d\theta \, d\phi \tag{6.40}$$

to obtain

$$W = \frac{A^2}{\rho_0 \, c_0} \left[1 \pm \frac{\sin k\, d}{k\, d} \right] \tag{6.41}$$

where the plus sign refers to an in-phase doublet and the minus sign to an out-of-phase doublet.

Now there is a great deal of important information in these few equations. Plotting Eq. 6.41 in Fig. 6.9, we can examine the relative power of a doublet compared to a simple source. The top curve shows the power of an in-phase pair as a function of k d. When the two sources are close (compared with a wavelength) together, k d is less than 1.

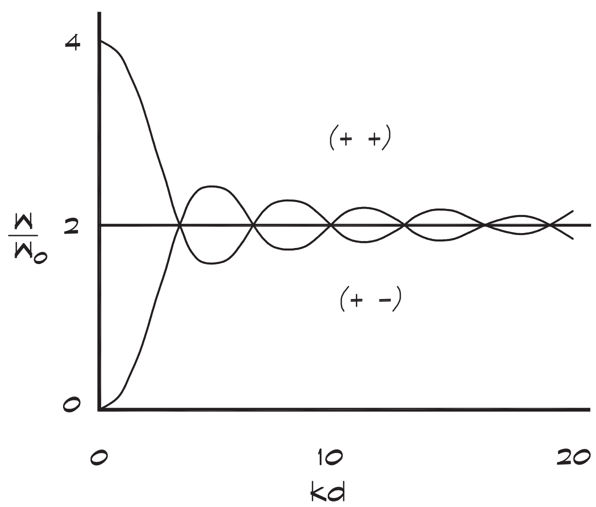
Recall that the wave number

$$k = \omega / c = 2\, \pi / \lambda \tag{6.42}$$

Therefore, when k d is less than 1, the separation distance d between sources is less than a sixth of a wavelength. For this configuration, the acoustic pressure effectively doubles, the combined source power increases by a factor of four, and the sound power level increases by 6 dB.

FIGURE 6.9 Total Power Radiated from a Doublet Source (Rossing and Fletcher, 1995)

The total power W of two omnidirectional sources having the same (+ +) or the opposite (+ −) polarity and separation distance d, as a function of the frequency parameter kd. The power radiated by a single source is W_0.



One way this can occur in buildings is when a source is placed close to a hard surface such as a concrete floor or wall. The surface acts like an acoustic mirror and the original source energy is reflected, as if the source were displaced by a distance d/2 behind the surface of the mirror. If the distances are small enough and the frequency low enough, the pressure radiates from the original and reflected source in phase in all directions. When sound level measurements are made very close to a reflecting surface, a 6 dB increase in level can be expected due to pressure doubling.

As the distance d between each source increases, the angular patterns become more complicated. The radiated power for a doublet at higher values of k d, also shown in Fig. 6.9, is about twice the power of a single source whether or not the sources are in phase. This is the same result that we found for incoherent (random phase) sources—a 3 dB (10 log N) increase for two sources combined.

Although the overall power of a doublet has a relatively simple behavior, the directivity pattern is more complicated. Typically, these directivity patterns are displayed in the form of polar plots. The front of the source, which is usually the loudest direction, is shown pointing toward the top of the diagram and the decrease in level with angle is plotted in increments around the source. A uniform directivity pattern is a perfect circle. The *directional characteristic* R_θ is one commonly encountered descriptor. It is the terms in the parentheses in Eq. 6.37 and represents the directional pattern of the sound pressure for a doublet. The directivity is its square and the directivity index is 10 log of that. The directional characteristic pattern produced by an in-phase doublet at various frequencies is shown in Fig. 6.10. Note that the direction of greatest level, when θ is 0, is at right angles to the line between the sources. The directivity patterns vary with frequency. The half-beamwidth angle, which is defined as the angle between on axis and the first zero, occurs when $(k\,d/2)\sin\theta = \pi/2$. The half beamwidth for an in-phase doublet is

$$\psi = \sin^{-1}(\lambda/2\,d) \tag{6.43}$$

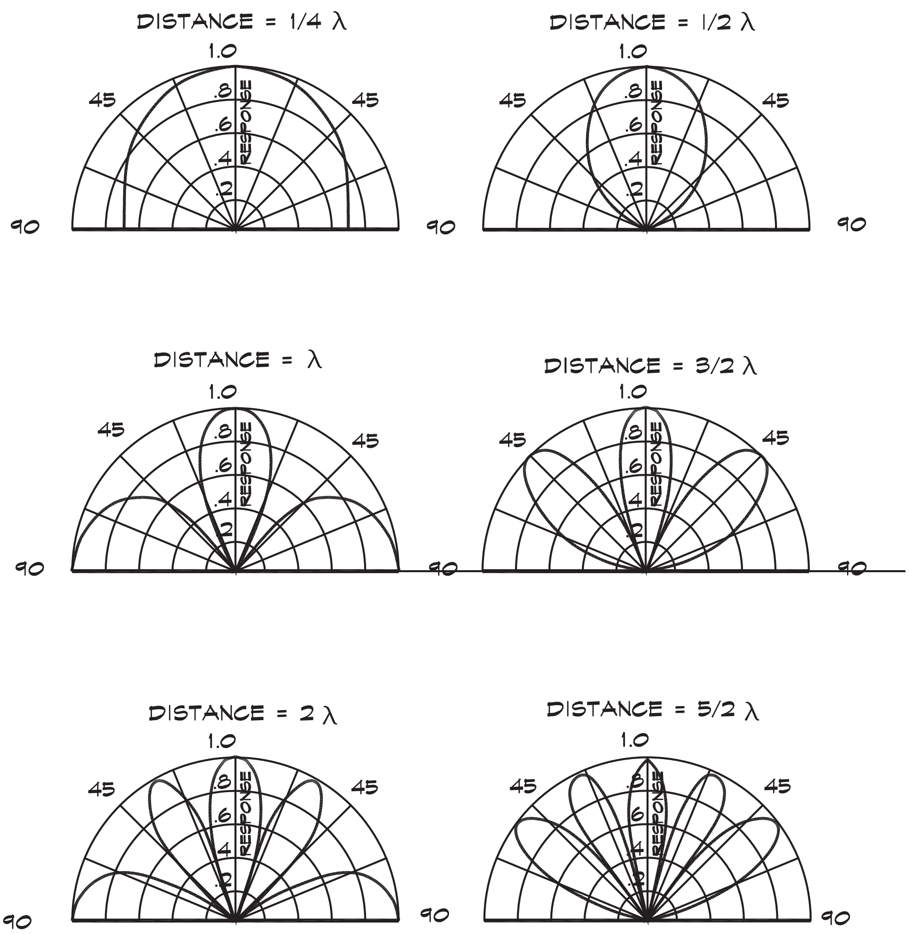
Dipole Sources and Noise Cancellation

If we have a doublet, where the two sources have opposite polarities, the configuration is called a dipole. A practical example of this type of source is an unbaffled loudspeaker. Since sound radiates from the rear of the loudspeaker cone as well as from the front, and the two signals are out of phase, the signal from the rear can combine with the front signal and produce a null pattern at right angles to the axis of the cone.

If the dipole sources are close together, when $k\,d < 1$, from Fig. 6.9 we see that the total power radiated approaches zero. This is the basis for the field of active noise cancellation in a three-dimensional acoustic space. Two sources of opposite polarity, when positioned close enough together, radiate a combined null signal. In practice, this can be accomplished by generating a cancellation signal quite close (d less than $\lambda/6$) to the source or to the receiver. A microphone can be used to sense the primary noise signal and by appropriate processing a similar signal having the opposite phase can be produced. Active noise cancellation systems are available in the form of headphones, which suppress sounds having a frequency of less than about 300–400 Hz. Source-cancellation systems for environmental noise control are less common, but have been applied successfully to large transformers and exhaust stacks. Noise cancellation systems have not been applied to the general run of noise problems, due

FIGURE 6.10 Directional Characteristics in Terms of R_θ of an In-Phase Doublet Source as a Function of the Distance between the Sources and the Wavelength (Olson, 1957)

The direction corresponding to the angle θ is measured relative to the perpendicular to the line connecting the two sources. The three dimensional polar plots are surfaces of revolution about the line joining the two sources.

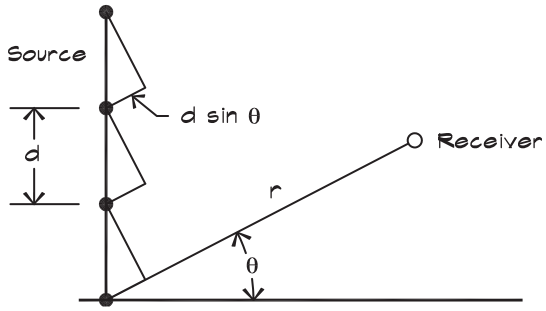


to the distance requirements outlined earlier, although one-dimensional problems such as duct-borne noise have been treated successfully. Due to the one-dimensional nature of ducts at low frequencies, the distance requirements for active noise control are not the same as they are for three-dimensional spaces.

Arrays of Simple Sources

When several simple sources are arrayed in a line, the phase relationship between them increases the directivity of the group along the $\theta = 0$ axis, shown in Fig. 6.11. Loudspeaker systems so configured are called line arrays and are a common arrangement

FIGURE 6.11 The Geometry of a Line Array



in loudspeaker cluster design. If n in-phase sources are equally spaced along a line, we can calculate the pressure in the far field following the same reasoning we used to derive Eq. 6.37. The pressure is

$$\mathbf{p} = \frac{n\,\mathbf{A}\,e^{j\omega t}\,e^{-jkr}}{r}\left[\frac{1}{n}\sum_{m=0}^{n-1} e^{jkmd\,\sin\theta}\right] \tag{6.44}$$

and the summation can be expressed in terms of trigonometric functions

$$\mathbf{p} = \frac{n\,\mathbf{A}\,e^{j\omega t}\,e^{-jkr}}{r}\left[\frac{\sin\left(\dfrac{n\,\pi\,d}{\lambda}\,\sin\theta\right)}{n\,\sin\left[\dfrac{\pi\,d}{\lambda}\,\sin\theta\right]}\right] \tag{6.45}$$

The term on the left of the brackets in Eq. 6.45 is the source strength for all n simple sources. At $\theta = 0$ the bracketed term goes to one so that the overall source strength is the same, as we would expect from a coherent group.

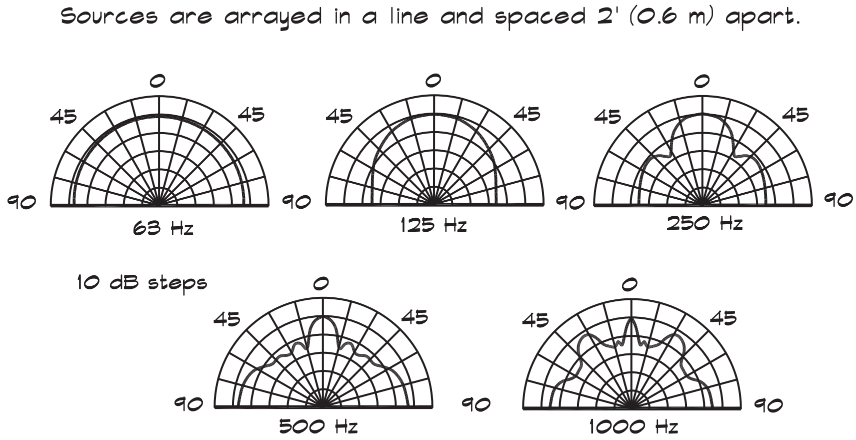
The polar patterns in terms of the directivity index for an array of four omnidirectional sources are shown in Fig. 6.12. The spacing is 2 ft (0.6 m) between sources. The angle to the point where the sound level is down 6 dB from the on-axis value occurs when the bracketed term in Eq. 6.45 equals 1/2, which must be solved numerically.

The half-beamwidth angle is more easily determined

$$\psi = \sin^{-1}\left[\frac{2\,\pi}{n\,k\,d}\right] = \sin^{-1}\left[\frac{\lambda}{n\,d}\right] \tag{6.46}$$

so that, when the total length of the line array is about 1.4 wavelengths, the first zero is at about $\pi/4$ radians (45°). Hence, to begin to achieve appreciable control over the beamwidth, line arrays must be at least 1.4 wavelengths long. For narrower coverage angles, the length must be longer. The region of pattern control, without undue lobing, ranges from an overall array length of about 1.4 wavelengths to the point where the spacing between elements approaches a wavelength.

FIGURE 6.12 Directivity Index of Four Point Sources



This has profound architectural consequences because it says that loudspeaker systems must be large in the vertical direction to control directivity in a vertical plane. For example, to limit the beamwidth angle to $\pi/3$ radians (60°) at 500 Hz, the array should be about 750 mm (30 inches) high. In cabinet systems the horn, which emits the 500 Hz signal, must be about 30 inches high to achieve a 40° vertical coverage angle. (Note that the coverage angle, which is the angle between −6 dB points, is less than the line array beamwidth.) Architecturally this means that in large rooms such as churches and auditoria, which often require highly directional loudspeaker systems to achieve adequate intelligibility, a space at least 4 feet (1.2 m) high must be provided for a speech reinforcement system. If live music is going to be miked, the directivity should extend down an octave lower to reduce feedback. This requires a line array about 8 feet long, similar to that shown in Fig. 6.12 in a concert venue, or significant loudspeaker displacement or barrier shielding in a permanent installation.

Line source configurations can also be used to control low-frequency directivity in concert systems. When concert loudspeaker systems are arranged by unloading truckloads of multiway cabinets and stacking them up on either side of the stage, there is little control of the low-frequency energy and extremely loud sound levels are generated near the front of the stage and at the performers. If instead, low-frequency cabinets (usually dual 18-inch woofers) are stacked vertically to a height of 20 to 30 feet (6 to 9 m), a line source is constructed that controls bass levels at the stage apron. The front row seats are at an angle of nearly 90° to the midpoint axis of the line source so even coverage is maintained from the front to the back of the seating area.

Continuous Line Arrays

The continuous line array is a convenient mathematical construct for modeling rows of sources that are all radiating in phase. Line arrays have a relatively narrow frequency range over which they maintain a simple directivity pattern. If their length is less than a half-wavelength, they will not provide appreciable directional control. At high frequencies line sources have a very narrow beamwidth, so that off-axis there can be a coloration of the sound.

The directional characteristic of a coherent line source can be obtained (Olson, 1957) by substituting $l \cong n\,d$ into Eq. 6.45. This approximation is true for large n.

$$R_\theta = \frac{\sin\left(\dfrac{\pi l}{\lambda}\sin\theta\right)}{\dfrac{\pi l}{\lambda}\sin\theta} \tag{6.47}$$

where R_θ is the directional characteristic of the sound pressure relative to the on-axis sound pressure, and l is the length of the line source.

The directivity plots are shown in Fig. 6.13. Practical considerations limit the size of a loudspeaker array to an overall length of about λ to 4 λ or so. This two-octave span is adequate for many sound source applications, where horns are used on the high end and line arrays are used for the midrange. Directional control is seldom required below the 250 Hz octave band except in concert venues.

As sources are added to a line array, the beamwidth decreases and the number of lobes increases. To compensate for this effect, a line source can be tapered by decreasing the level of the signal fed to loudspeakers farther from the center. The directional characteristic (Olson, 1957) of a tapered line source, whose signal strength varies linearly from the center to zero at the ends is

$$R_\theta = \frac{\sin^2\left(\dfrac{\pi l}{\lambda}\sin\theta\right)}{\left(\dfrac{\pi l}{\lambda}\sin\theta\right)^2} \tag{6.48}$$

As Fig. 6.14 shows, tapering broadens the center lobe of the directivity pattern and decreases the off-axis lobing and the expense of overall sound power.

Curved Arrays

Loudspeakers can be configured in other ways, including convex or concave arcs, twisted line arrays, or helical line sources, which look like a stack of popsicle sticks. For a series of sources arranged in a curve the directivity pattern in the plane of the arc is (Olson, 1957)

$$R_\theta = \frac{1}{2n+1}\left\{\sum_{m=-n}^{m=n}\cos\left[\frac{2\pi r_a}{\lambda}\cos(\theta + m\phi)\right] + j\sum_{m=-n}^{m=n}\sin\left[\frac{2\pi r_a}{\lambda}\cos(\theta + m\phi)\right]\right\} \tag{6.49}$$

where R_θ = directional characteristic of the array sound pressure relative to the on axis sound pressure

θ = angle between the radius to the center source and the line to the receiver, (rad)

$2n + 1$ = number of sources in the array

$j = \sqrt{-1}$

λ = wavelength, (m)

m = integer variable

ϕ = angle subtended by adjacent sources on the arc, (rad)

FIGURE 6.13 Directional Characteristics of a Coherent Line Source (Olson, 1957)

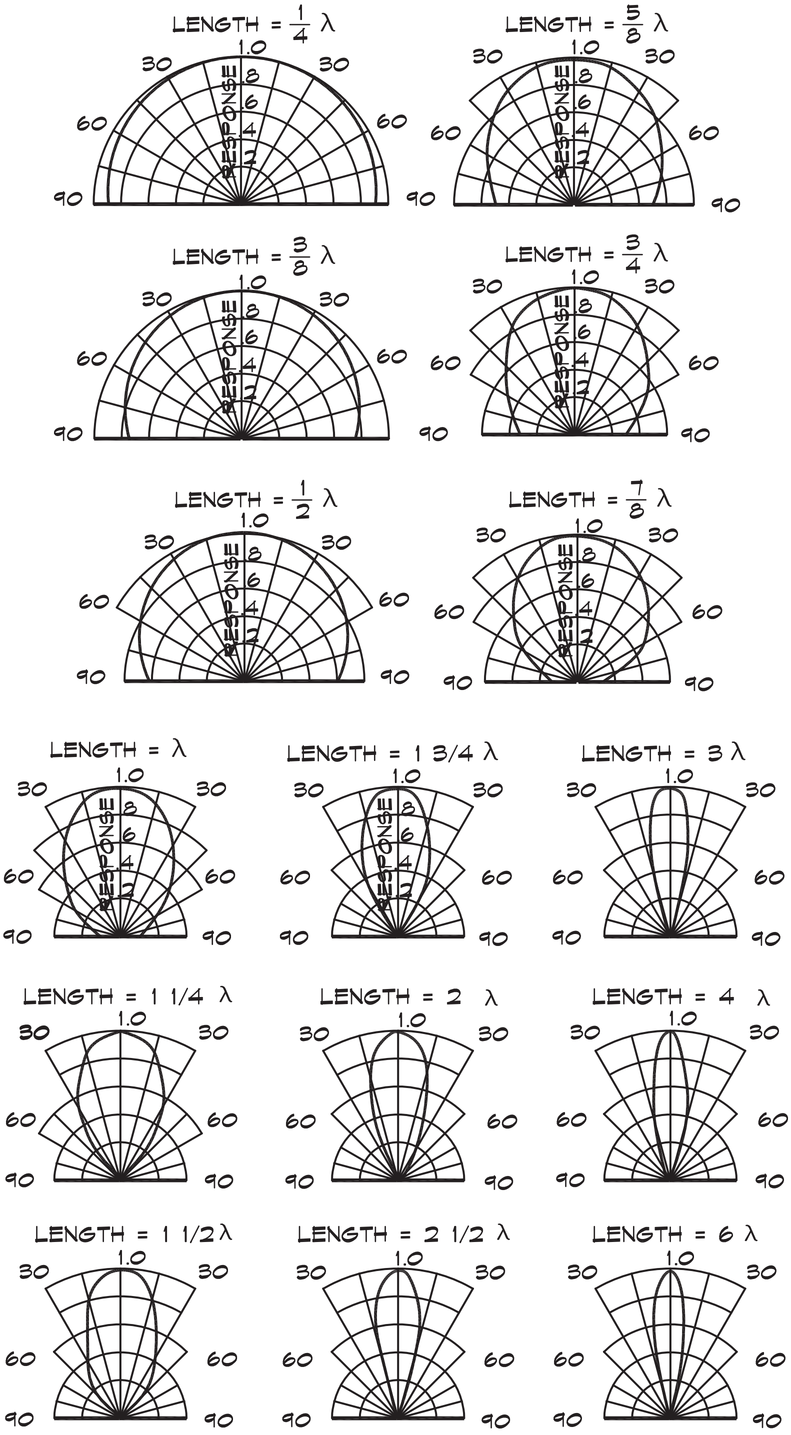


FIGURE 6.14 Directional Characteristics R_θ of a Tapered Line Source—Linear Taper (Olson, 1957)

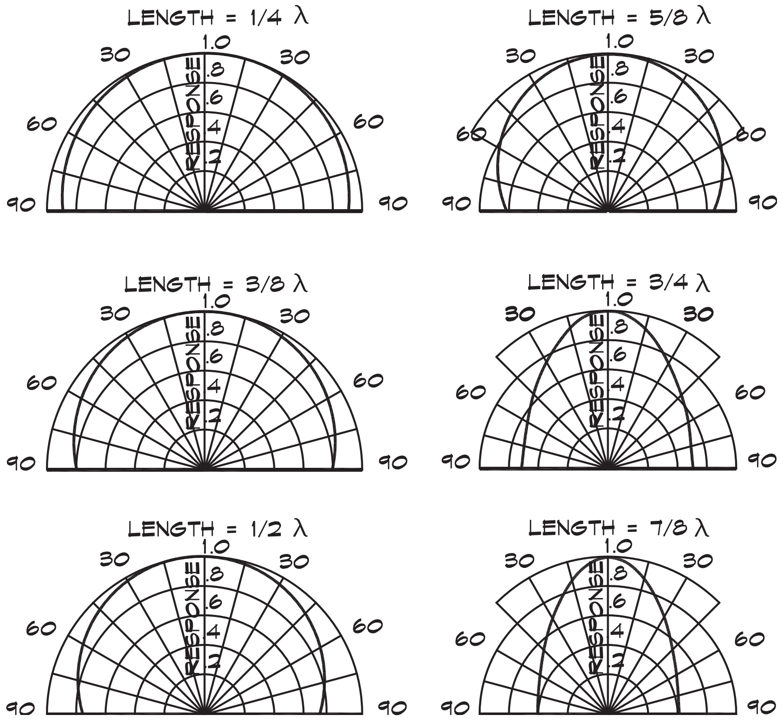
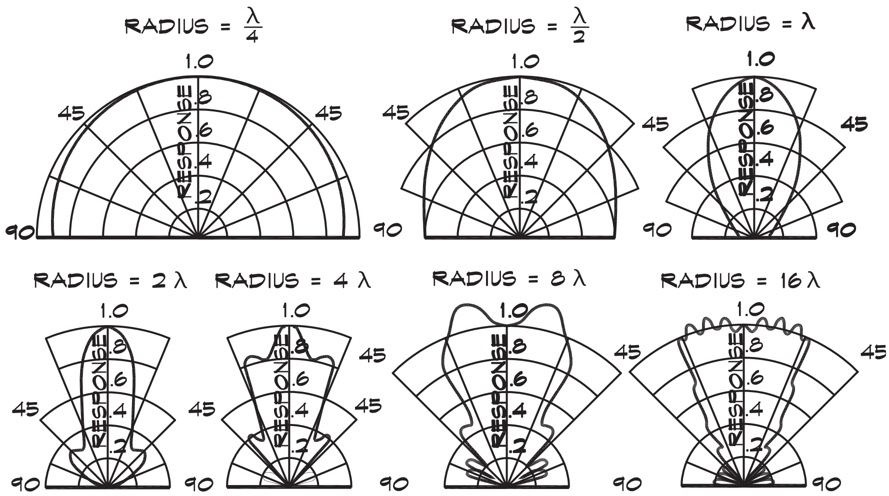


FIGURE 6.15 Directional Characteristics R_θ of a 60° Segment of Arc (Olson, 1957)



An example of the directivity pattern for a 60° arc is shown in Fig. 6.15. At very high frequencies, the directivity pattern starts to look like a wedge. This behavior was the basis for the design of multicellular horns, which were developed to reduce the high-frequency beaming associated with large horn mouths.

Phased Arrays

A phased array consists of a line or planar group of sound sources, which are fed an electronic signal such that the direction of the emitted wavefront can be steered by controlling the phase or time delay (n τ) to each source. The directional factor becomes (Kinsler et al., 1982)

$$R_\theta = \frac{1}{n}\left[\frac{\sin\left[\frac{n\,\pi\,d}{\lambda}\left(\sin\theta - \frac{c\,\tau}{d}\right)\right]}{\sin\left[\left(\frac{\pi\,d}{\lambda}\,\sin\theta - \frac{c\,\tau}{d}\right)\right]}\right] \tag{6.50}$$

This technique may be used to electronically direct signals radiated from a line or panel of sources. It also can be used to detect the direction of an incoming signal incident on a line of microphones by sensing the time delay between transducers. The major lobe is pointed in a direction given by

$$\sin\,\theta_0 = \frac{c\,\tau}{d} \tag{6.51}$$

which is independent of frequency.

Source Alignment and Comb Filtering

When two sources are separated by a distance d, and a receiver lies at an angle θ to their common axis, as shown in Fig. 6.8, there is a difference in distance from each source to the receiver. This difference is d sin θ so depending on the frequency of the sound radiating from the doublet the signals may be in or out of phase. If the path length difference is an even multiple of a wavelength then the signals will add and the composite signal will be 6 dB higher. If the path length difference is an odd multiple of a half wavelength then the signals will cancel and the composite signal will be a null. The consequence of this is that as we sweep across a range of frequencies the doublet source will generate a series of filters whose maximum frequencies are given by

$$f_n = \frac{n\,c}{d\,\sin\theta} \tag{6.52}$$

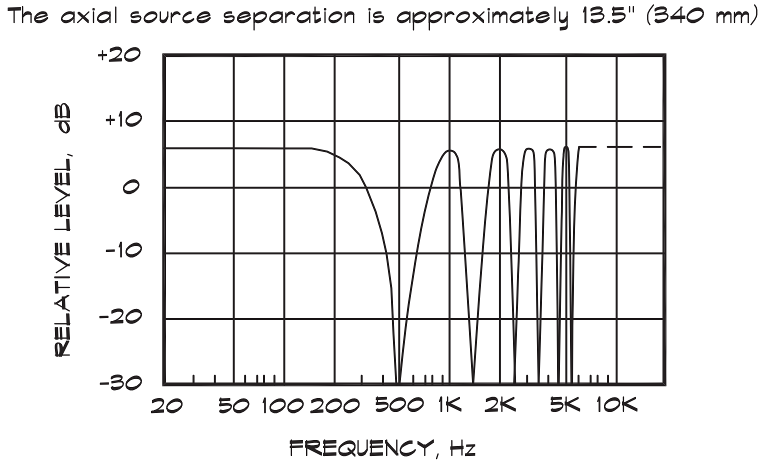
where the number n is an integer that ranges from one to infinity, or at least to the upper frequency limit of audibility. The null frequencies are given by

$$f_n = \frac{(2n-1)\,c}{2\,d\,\sin\theta} \tag{6.53}$$

At $\theta = \pi / 4$ (45°) and a doublet separation distance of 0.5 m (1.6 ft), the null frequencies are 486, 1458, 2430, . . . Hz.

If a broadband signal such as speech is transmitted by means of the doublet source, the resultant signal shown in Fig. 6.16 displays a series of dips, which are shaped much like the teeth of a comb (hence the moniker, comb filter). Deep notches at each of these frequencies can have a negative influence on the clarity of the received signal. The extent of the influence depends on the separation between frequencies and the depth of the nulls. The effect can be corrected somewhat by introducing an electronic delay; however, the delay is exact for one direction only. A loudspeaker design strategy, which reduces the time difference between

FIGURE 6.16 The Comb Filter Produced by Two Sound Sources (Everest, 1994)

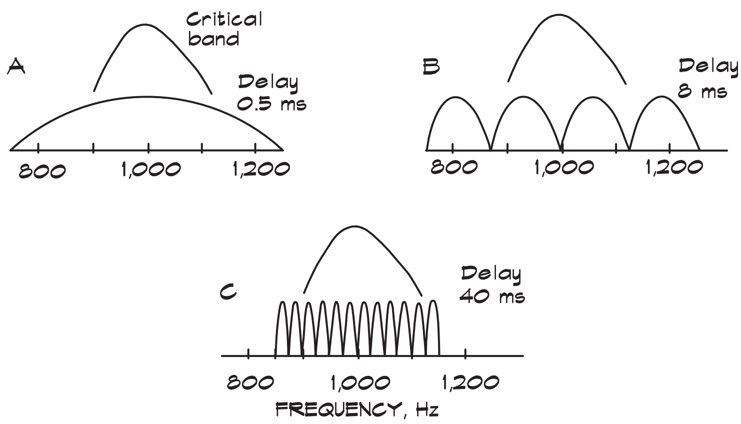


components, reduces the comb filtering effect. It is best to match the time delays at points where the two signals have nearly equal amplitudes. If one source is substantially (6 dB or so) louder than a second, the comb filtering effects are much less.

Comb Filtering and Critical Bands

Everest (1994) provides an interesting analysis of the audibility of comb filtering, which is illustrated in Fig. 6.17. The perceptual importance of comb filtering can be understood in terms of the effective bandwidth of the individual filter, compared to the width of a critical band. When the time delay between sources is small, say 0.5 ms, the range of frequencies between nulls is quite broad—much greater than the width of a critical band at 1000 Hz, which is about 128 Hz. Therefore, the effect of the delay is perceptible. When the time delay is large, say 40 ms, there are many nulls within one critical band, and the effects are integrated by the ear and are not perceptible. This helps explain why comb-filtering effects are not a problem in large auditoria, where reflections off a wall frequently create a delayed

FIGURE 6.17 Audibility of Comb Filtering (Everest, 1994)



signal of 40 ms or more, without appreciably coloring the sound. In loudspeaker clusters and small studios, small loudspeaker misalignments and reflections from nearby surfaces can be quite noticeable.

6.4 COHERENT PLANAR SOURCES

Piston in a Baffle

The physical effects of sound sources of finite extent are described using a few simple models, which can be applied to a wide range of more complicated objects. The most frequently utilized example is the piston source mounted in a baffle, which was first analyzed by Lord Rayleigh in the nineteenth century. The baffle in this example is an infinite solid wall. The piston may be a loudspeaker or simply a slice of air with all portions of the slice moving in phase at a velocity $\mathbf{u} = u_0 \, e^{j \omega t}$. The piston is located at the origin, on the surface of the wall, and pointed along the r axis as shown in Fig. 6.18. The sound pressure at a distance r due to a small element of surface on the piston is

$$\mathbf{p}(r, \theta, t) = j \frac{\rho_0 \, c_0 \, u_0 \, k}{2 \pi} \int_s \frac{e^{j (\omega t - k r')}}{r'} ds \tag{6.54}$$

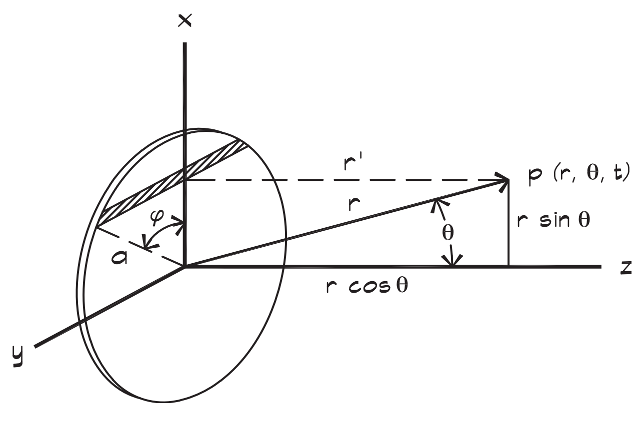
If we divide the surface into horizontal slices the incremental pressure due to a slice dx high by $2 \, a \sin \phi$ wide is

$$d\mathbf{p} = j \, \rho_0 \, c_0 \frac{u_0}{\pi r'} k \, a \, \sin \phi \, e^{j (\omega t - k r')} dx \tag{6.55}$$

In the far field where r >> a we use the approximate value for r'

$$r' \cong r \left(1 - \frac{a}{r} \sin \theta \, \cos \phi\right) \tag{6.56}$$

FIGURE 6.18 **Geometry of a Piston in a Baffle (Kinsler et al., 1982)**



The pressure is then calculated by integrating over the surface of the piston.

$$\mathbf{p} = j\, \rho_0\, c_0\, \frac{u_0}{\pi r}\, k\, a\, e^{j\,(\omega t - k r)} \int_{-a}^{a} e^{j k a \sin\theta \cos\phi} \sin\phi\, dx \tag{6.57}$$

This can be done by making the substitution $x = a \cos\phi$ and integrating over the angle (see Kinsler et al., 1982).

$$\mathbf{p} = \frac{j\, \rho_0\, c\, u_0\, k\, a^2}{2\, r}\, e^{j\,(\omega t - k r)} \left[\frac{2\, J_1\, (k\, a\, \sin\theta)}{(k\, a\, \sin\theta)} \right] \tag{6.58}$$

where θ = angle between the center axis and the line to the receiver, (rad)
J_1 = Bessel function of the first kind
$j = \sqrt{-1}$
λ = wavelength, (m)
ω = radial frequency, (rad/s)
t = time, (s)

The Bessel function produces a number, which depends on an argument, similar to a trigonometric function. It is the solution to a particular type of differential equation and can be calculated from an infinite series of terms, which follow the pattern

$$J_1\, (x) = \frac{x}{2} - \frac{2\, x^3}{2 \cdot 4^2} + \frac{3\, x^5}{2 \cdot 4^2 \cdot 6^2} - \cdots \tag{6.59}$$

The resulting directional characteristic for a piston in a baffle is given in Fig. 6.19. The pattern in the far field depends on the ratio of the circumference of the piston to the wavelength of sound being radiated, which is the term (k a).

The directivity relative to the on-axis intensity is

$$Q_{rel} = \left[\frac{2\, J_1\, (k\, a\, \sin\theta)}{(k\, a\, \sin\theta)} \right]^2 \tag{6.60}$$

and the beamwidth as defined by the angle between the −6 dB down points on each side occurs at

$$Q_{rel} = .25 \tag{6.61}$$

This defines (Long, 1983) the relationship between the coverage angle (between the −6 dB points) and the piston diameter as

$$k\, a\, \sin\theta = 2.2 \tag{6.62}$$

When the piston diameter, 2 a, is equal to a wavelength the coverage angle $2\,\theta \cong 90°$. This is a useful rule of thumb in loudspeaker and horn design.

FIGURE 6.19 Directional Characteristics of a Circular Piston in a Baffle (Olson, 1957)

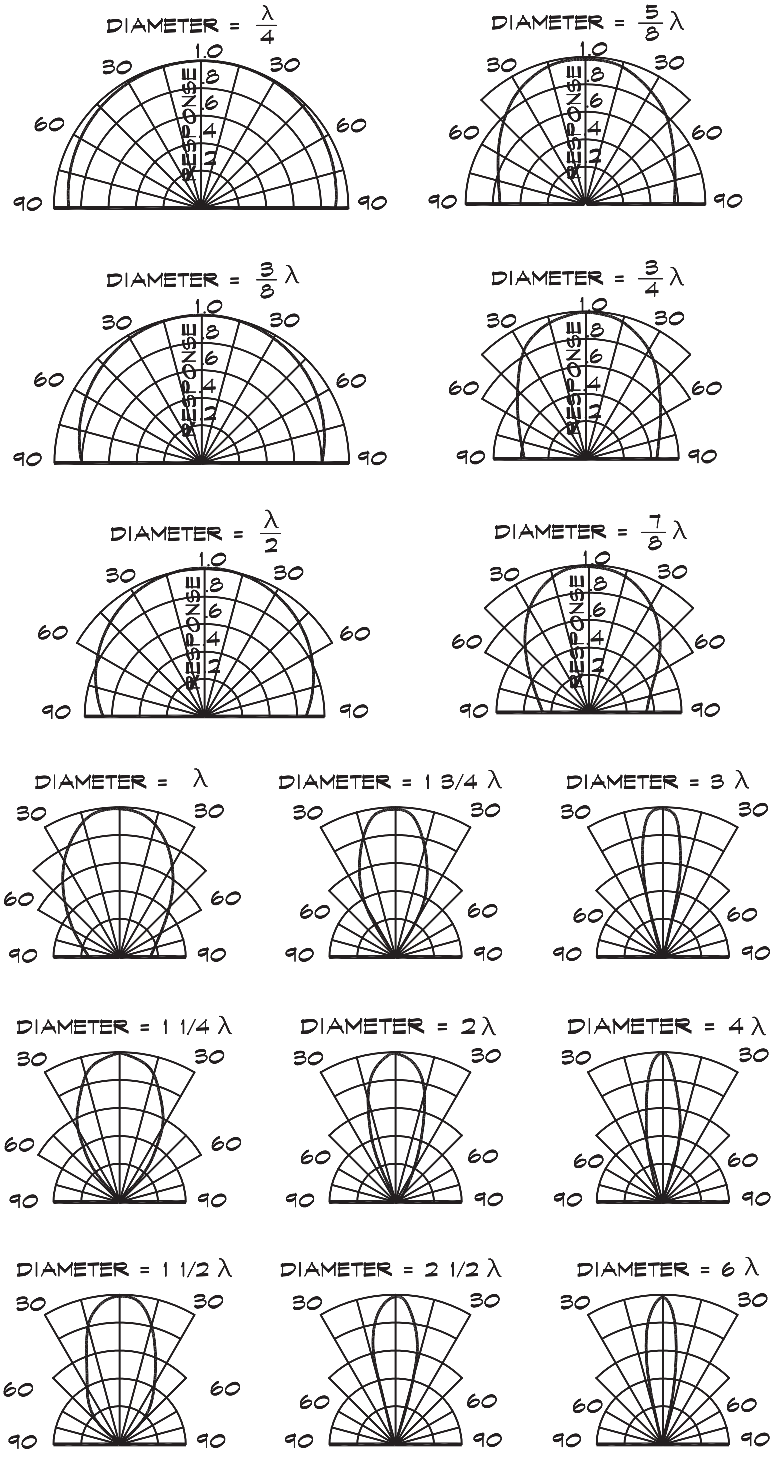
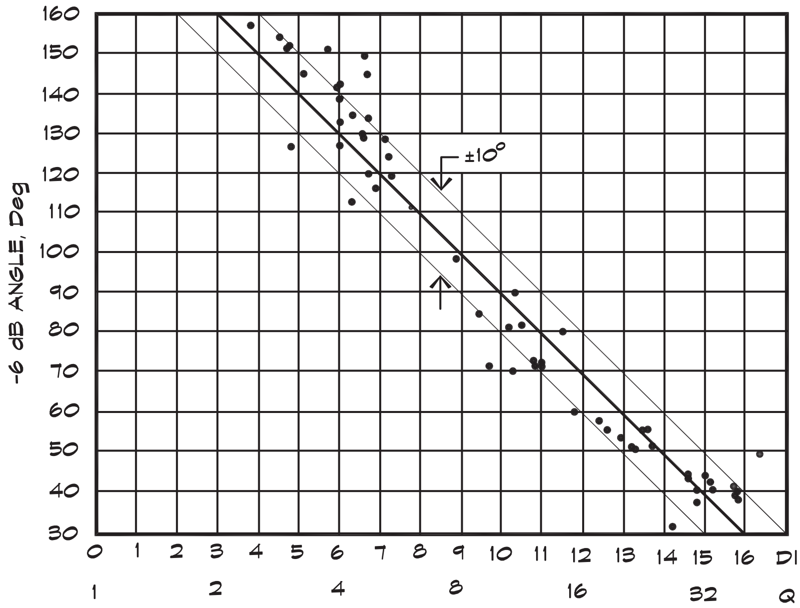


FIGURE 6.20 Relationship of Directivity Index to −6 dB Angle for Many Different Cone Type Loudspeakers and Baffles (Henricksen, 1980)



Coverage Angle and Directivity

Henricksen (1980) has compiled a chart of the coverage angle, which is defined as the included angle between the −6 dB down points on a polar plot, versus the source directivity for various sizes of cone loudspeakers. It is reproduced as Fig. 6.20. Referring to this figure, a coverage angle of 90° is equivalent to a Q of about 10. Therefore, if a cone loudspeaker or a horn is to achieve significant directional control, its mouth must be at least a wavelength long in the plane of the coverage angle. Typical directivity patterns for various sizes of cone loudspeakers and baffles are shown in Fig. 6.21 (Henricksen, 1980). The low-frequency directivity is determined by the size of the baffle, which sets the point at which the loudspeaker-baffle combination reverts to a point source with 360° coverage. Henricksen generalized the expected behavior from various loudspeaker and baffle sizes in Fig. 6.22. Here the high-frequency directivity is controlled by cone size, with the smaller cones being less directional. It is interesting to note that in home and near-field monitor loudspeakers, where wide dispersion angles are desired, the crossover to a smaller loudspeaker is made when the wavelength is equal to the cone diameter ($k\ a = \pi$). In sound reinforcement systems, where pattern control is of paramount importance, the crossover is made to a larger format driver when the wavelength is equal to a diameter. This is one reason why monitor loudspeakers are usually not appropriate for commercial sound reinforcement applications and why these systems can be made much smaller than commercial systems.

Loudspeaker Arrays and the Product Theorem

When an array of identical loudspeakers is constructed, the composite directivity pattern is determined by the directional characteristics of the array as well as the inherent directivity of the loudspeakers. The relationship between these directivities is given by the product theorem, which states that the overall directivity of an array of identical sources is the product

FIGURE 6.21 A Chart for Predicting the −6 dB Angle Response for Piston and Box Systems (Henricksen, 1980)

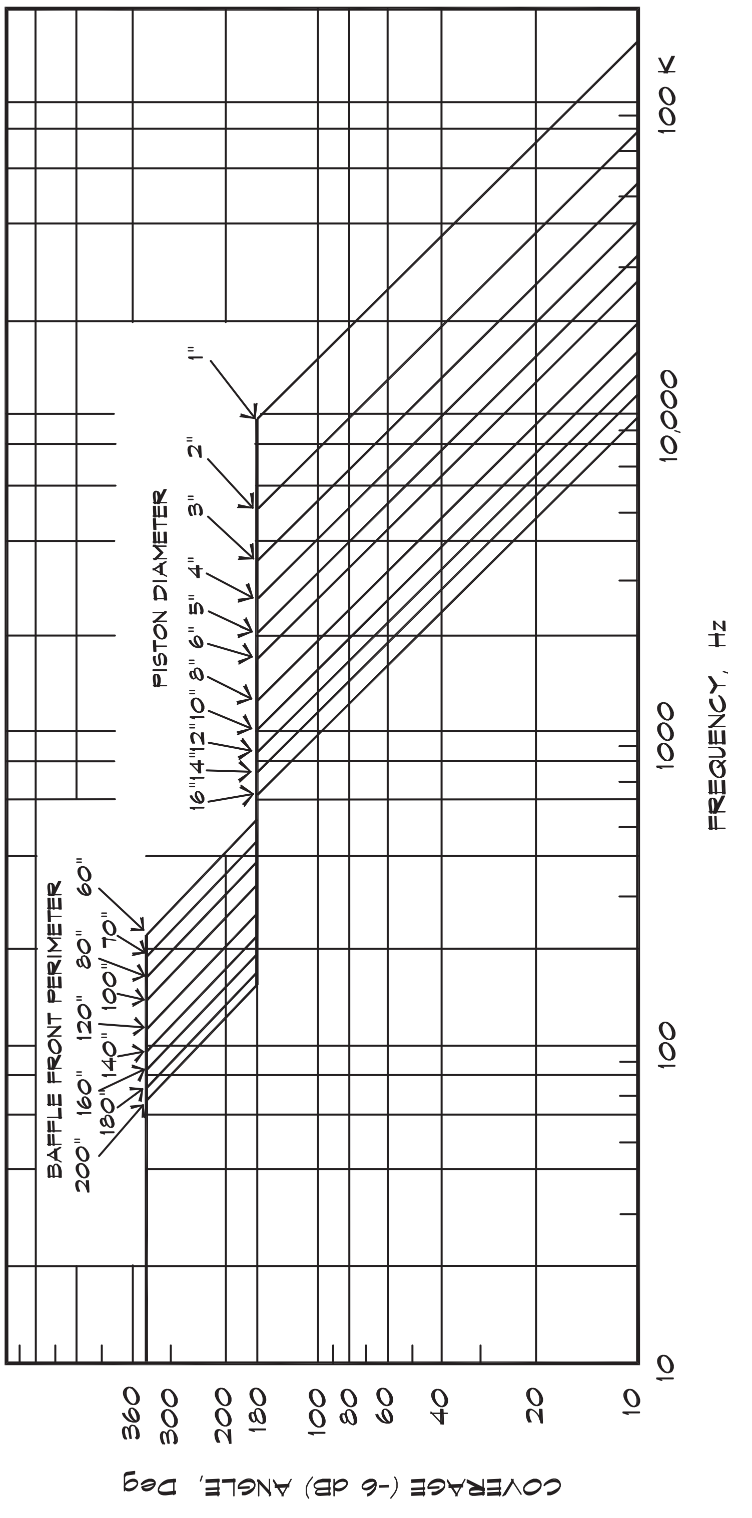
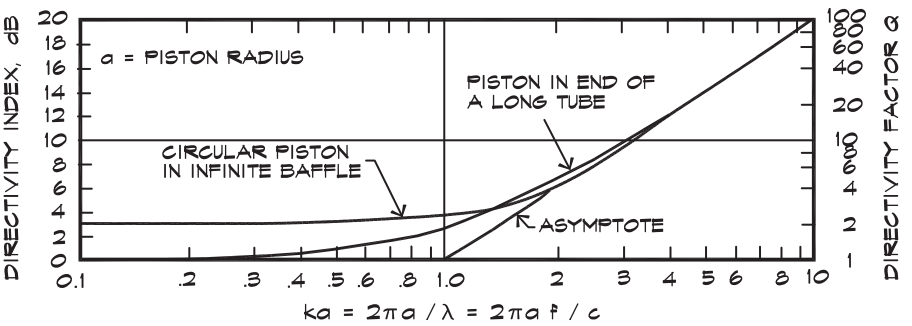


FIGURE 6.22 Directivity Index Response of Direct-Radiator Pistons in Various Configurations (Beranek, 1954)



of the directivities of the individual sources and the directivity due to the array.

$$Q_\theta\,(\theta,\,\phi) = Q_0\,Q_{rel}\,(\theta,\,\phi)\,R_\theta^2\,(\theta,\,\phi) \tag{6.63}$$

where $Q_\theta\,(\theta,\,\phi)$ = overall array directivity

Q_0 = on - axis directivity for an individual speakers

$Q_{rel}\,(\theta,\,\phi)$ = off - axis directivity of a given loudspeaker

$R_\theta^2\,(\theta,\,\phi)$ = array directivity relative to the on - axis sound intensity

If the array is composed of different types of loudspeakers or loudspeakers at various levels, for which there is no ready directional characteristic, the directivity must be calculated for each element of the array from its off-axis level and the relative phase due to its position in the array. The off-axis phase behavior for individual loudspeakers may also be important but is seldom available in the data published by loudspeaker manufacturers. Computer programs that perform these calculations are commercially available.

There are theoretical directivity values published for arrays that have a tapered volume level or other more exotic shadings. Olson (1957) has given a general equation for the directivity of a linear array having an arbitrary taper. Davis and Davis (1987) have published a scheme for improving the directivity of an array of loudspeakers in a baffle using a Bessel function weighting. Line arrays have been designed using axially rotated elements to disperse the high frequencies or vertically angled elements to smooth the frequency response. In these arrayed systems, it usually is assumed that each element radiates individually without influence from the others. For this assumption to hold true each transducer should be housed in a separate enclosure so that the loading produced by back radiation into a common cabinet does not influence the other loudspeakers.

Rectangular Pistons

The directional characteristic of a coherent rectangular piston source can be calculated in a similar fashion to that used for the circular piston in a baffle. McLachlan published these calculations in 1934. He showed that a rectangular rigid plate of length 2 a and height 2 b, which vibrates in an infinite rigid baffle, generates a far field directional characteristic

$$R_{\theta,\phi} = \left\{ \left[\frac{\sin\,(k\ a\ \cos\,\phi)}{(k\ a\ \cos\,\phi)} \right] \left[\frac{\sin\,(k\ b\ \cos\,\theta)}{(k\ b\ \cos\,\theta)} \right] \right\} \tag{6.64}$$

The term in the second set of brackets is the same as that found for a line source in Eq. 6.47. In fact, the rectangular piston has the same directional characteristic as the product of two line sources of length 2a and 2b arranged at right angles to one another, whose strength and phase are the same. The rectangular mouth of a horn loudspeaker can be modeled using this equation.

Force on a Piston in a Baffle

The impedance seen by a piston in a baffle can be obtained from Eq. 6.54; in this case the integral is evaluated close to the piston rather than in the far field. The pressure varies across the face of the piston but the quantity of interest is the force on the whole piston. This, along with the piston velocity, yields the mechanical radiation impedance seen by the piston (see Morse, 1948 or Kinsler et al., 1982).

$$\mathbf{z}_{\mathrm{r}} = \int \frac{\mathrm{d}\mathbf{F}_{\mathrm{S}}}{\mathbf{u}} = \rho_0\, c_0\, S\left[w_{\mathrm{r}}(2ka) + j\, x_{\mathrm{r}}\,(2ka)\right] \tag{6.65}$$

The integration of the force over the face of the piston is complicated and will not be reproduced in detail. The impedance terms include a real part

$$w_r = 1 - \frac{J_1(2k\,a)}{k\,a} = \frac{(k\,a)^2}{2} - \frac{(k\,a)^4}{2^2\cdot 3} + \frac{(k\,a)^6}{2^2\cdot 3^2\cdot 4} - \cdots \tag{6.66}$$

which has limiting values of

$$\begin{aligned} w_r &\rightarrow \frac{(k\,a)^2}{2} \quad \text{for } k\,a << 1 \\ &\rightarrow 1 \quad \text{for } k\,a >> 1 \end{aligned} \tag{6.67}$$

and an imaginary part

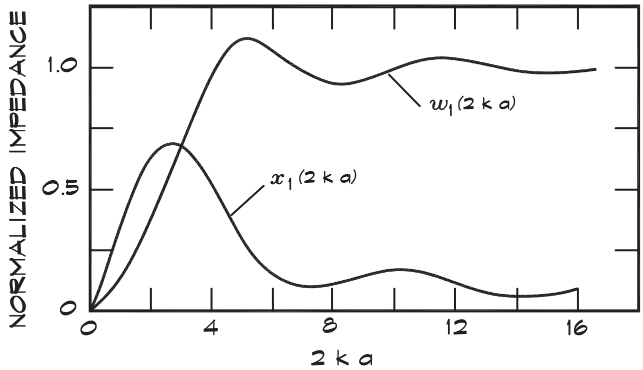
$$x_r = \frac{1}{\pi\,(k\,a)^2}\left[\frac{(2k\,a)^3}{3} - \frac{(2k\,a)^5}{3^2\cdot 5} + \frac{(2k\,a)^7}{3^2\cdot 5^2\cdot 7} - \cdots\right] \tag{6.68}$$

with limits

$$\begin{aligned} x_r &\rightarrow \frac{8\,k\,a}{3\,\pi} \quad \text{for } k\,a << 1 \\ &\rightarrow \frac{2}{\pi\,k\,a} \quad \text{for } k\,a >> 1 \end{aligned} \tag{6.69}$$

Figure 6.23 shows the resistive (real) and reactive (imaginary) components of the radiation impedance. At low frequencies, the reactive component dominates, whereas at high frequencies, the resistance becomes more important. The impedance at high frequencies approaches the area times $\rho_0\ c_0$. We will need these results in our later analysis of the absorption due to quarter-wave resonator tubes and quadratic-residue diffusers.

FIGURE 6.23 Impedance Functions of a Baffled Piston (Kinsler et al., 1982)



6.5 LOUDSPEAKERS

Cone Loudspeakers

A moving coil or cone loudspeaker, illustrated in Fig. 6.24, is the most commonly used type. It consists of a circular cone of treated paper or other lightweight material, which is attached to a coil of wire suspended in a permanent magnetic field. When a current passes through the wire, the coil is forced out of the magnetic field in one direction or another depending on the direction of the current. A sinusoidal voltage applied to the wire results in sinusoidal motion of the cone. Many broadband cone loudspeakers have a small dome or dust cap at their center, which helps disperse the high frequencies.

FIGURE 6.24 A Simple Moving Coil Loudspeaker (Kinsler et al., 1982)

A) Magnet, B) Voice Coil, C) Diaphragm, D) Corrugated Rim,
E) Spider (for stiffness), F) Dome

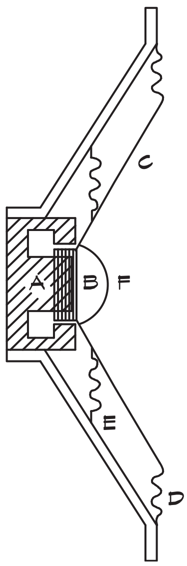
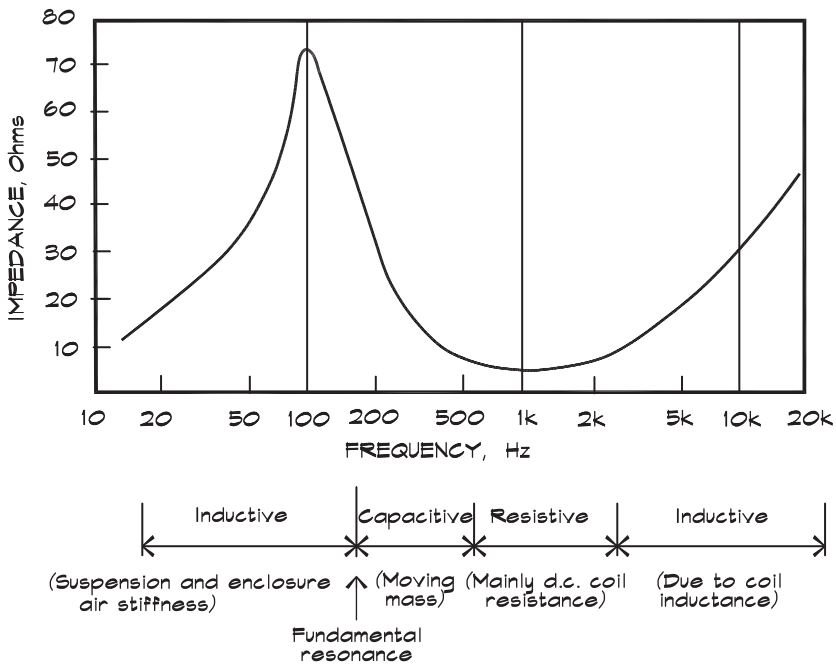


FIGURE 6.25 Input Impedance of a Moving Coil Driver (Colloms, 1980)



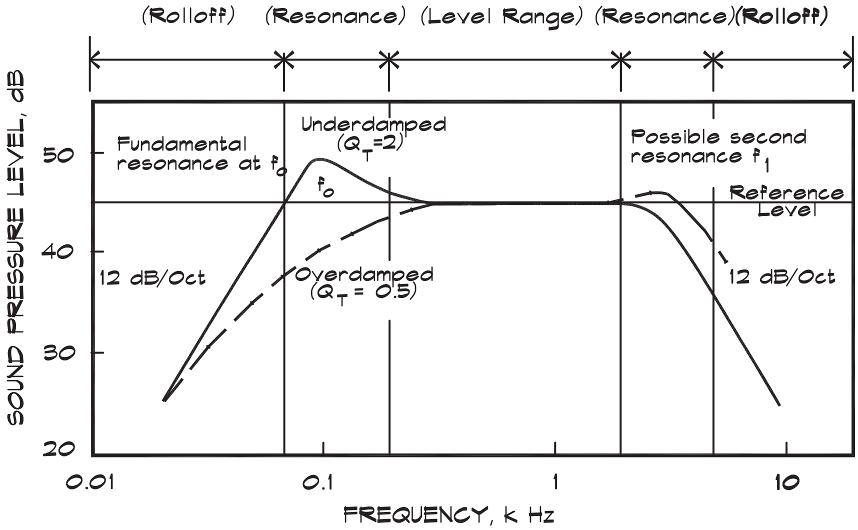
Cone loudspeakers are not particularly efficient sound radiators. Typically, they convert between 0.5 and 2% of the electrical energy to sound. A loudspeaker, driven with 1 electrical Watt, will radiate about 0.01 acoustical Watts of energy, which is equal to a sound power level of 100 dB. At a distance of 1 meter, such a loudspeaker would result in a sound pressure level of about 92 dB assuming a Q of two. This number, that is the on-axis level generated at 1 m for 1 W of power, is the sensitivity of the loudspeaker and was defined in Eq. 2.76.

Loudspeakers are characterized not only by their sensitivity but also by their impedance, frequency response, directivity, and polar pattern or coverage angle. Each of these parameters is useful to the designer. The electrical impedance is like the acoustical impedance in that it represents the electrical resistance of the loudspeaker, which is a complex number. Not all of the resistance is electrical. The mechanical impedance is reflected back as electrical impedance, which is not constant with frequency. Figure 6.25 gives a typical impedance curve for a loudspeaker in a baffle.

The low-frequency peak is at the fundamental resonance of the loudspeaker cone's spring mass system, including the spring effect of the air suspension system. Above the resonant frequency, the impedance drops to a region where it mainly consists of the dc resistance of the coil and reaches a minimum value, which can be measured with an ohmmeter. At the high- and low-frequency extremes, the impedance is mainly inductive, due to the stiffness of the suspension system at low frequencies, and to the coil inductance at high frequencies. Usually the minimum is listed by loudspeaker manufacturers since it is this value that controls the maximum current flow at a given applied voltage.

The frequency response of a loudspeaker is available from the manufacturer, and there is an example in Fig. 6.26. It is measured by sweeping a signal of constant voltage across the frequency range of interest and measuring the sound pressure level of the loudspeaker

FIGURE 6.26 Response of a Moving Coil Driver (Colloms, 1980)



on-axis at a given distance. At both high and low frequencies the response curve rolls off at about 12 dB per octave. In between, it is relatively flat.

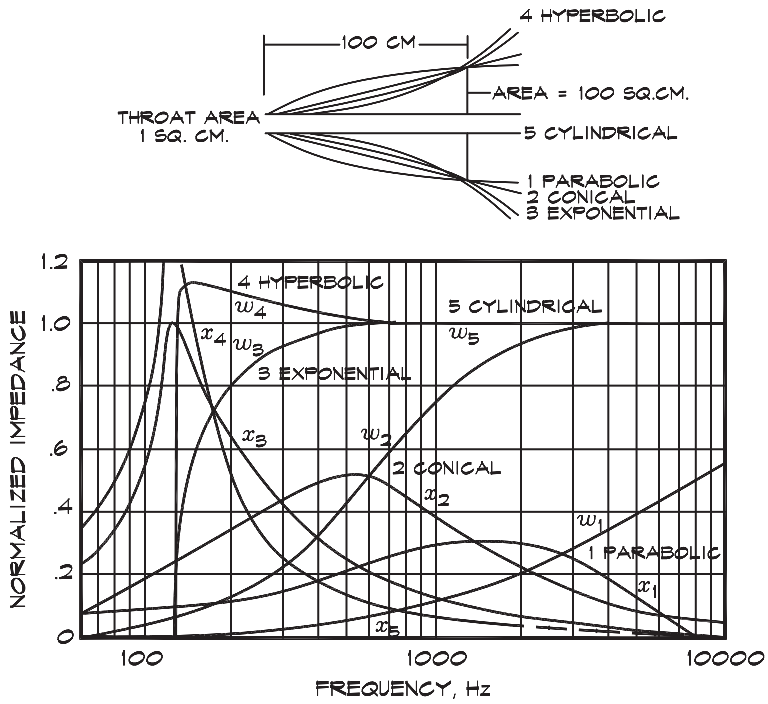
The low-frequency portion of the curve is influenced by the configuration of the enclosure. If the loudspeaker is not enclosed the cone radiates as a dipole, with the sound coming from the back canceling out the sound coming from the front. An infinite baffle reduces the dipole effect but is not always convenient to build. An enclosed box helps improve the low-frequency response by eliminating the dipole effect, but the air spring increases the resonant frequency of the cone. A ported enclosure acts as a second loudspeaker at low frequencies, which radiates in phase with the front of the cone. Due to the port resonance, the box emits more energy at low frequencies than the cone does.

Horn Loudspeakers

The use of horns undoubtedly originated with the cupping of the hands around the mouth to increase projected level. The Greeks applied the same idea when they attached conical horns to their theatrical masks to amplify the actors' voices. The development of brass instruments, at least as far back as the Romans, used a flared bell mouth. In the early twentieth century the exponential horn was studied and incorporated by Edison into his phonograph. The exponential horn shape was utilized in one form or another until the early 1980s, when the constant-directivity horn was introduced. Most loudspeaker manufacturers now offer a version of this type of horn.

A horn serves three functions. First, it provides an increased resistive loading for the driver so that it can work against a higher impedance than the air alone would present. Second, it improves the efficiency of the driver by constraining the air that is moved, gradually transitioning the air column into the surrounding space. Third, it controls the coverage pattern of the sound wave by providing side walls, which direct the beam of energy as it radiates away from the driver. Each of these functions may make conflicting demands on the horn designer. The shape that is the most efficient for impedance matching does not always provide the required directivity. The designer has to determine the functions the horn must perform, and trade off the advantages and disadvantages of each to get the best compromise solution.

FIGURE 6.27 Resistive and Reactive Components of the Normalized Acoustic Impedance of Infinite Horns (Olson, 1957)



The presence of a horn increases the impedance that is presented to the driver diaphragm. This causes the diaphragm to push against a higher pressure, which, although it makes the driver work harder, also allows more energy to be transmitted to the air. The resistive load placed on the diaphragm is almost totally dependent on the shape of the horn. Figure 6.27 plots the acoustical resistance of an infinitely long horn for five different shapes. As the diagram shows, a conical (straight-sided) horn does not add significant loading to the diaphragm. The cylindrical tube presents an even loading but has no increase in mouth area. The exponential horn, so called because the shape of its sides follows the exponential equation

$$S = S_0 \, e^{m x} \tag{6.70}$$

where S = horn area at distance x (m^2)
S_0 = throat area at x = 0 (m^2)
m = flare constant (m^{-1})
x = distance from the throat (m)

provides smooth loading down to a cutoff frequency

$$f_c = \frac{m\,c}{4\,\pi} \tag{6.71}$$

below which sound does not propagate without loss. Although the loading is constant with frequency for an exponential horn, the shape is not necessarily the best choice. For low-frequency directional control, the horn mouth size has to be relatively large, which leads to high-frequency control problems.

As we have seen from the piston in a baffle analysis, loudspeakers undergo a narrowing in their coverage pattern, called beaming, at high frequencies. To achieve wide coverage, high-frequency drivers must be physically small. Small drivers, however, are inefficient since they neither travel very far, nor push much air. Coupling a small driver to a horn helps solve both the beaming and the efficiency problems. Horn efficiencies as high as 50% can be achieved over a narrow range of frequencies; however, for broadband signals an efficiency of 10% is more likely. The sensitivity of a typical large format horn/driver is about 113 dB, which, with an on-axis Q of 20, represents an efficiency of a little more than 13% or an acoustic power of about 0.13 Watts.

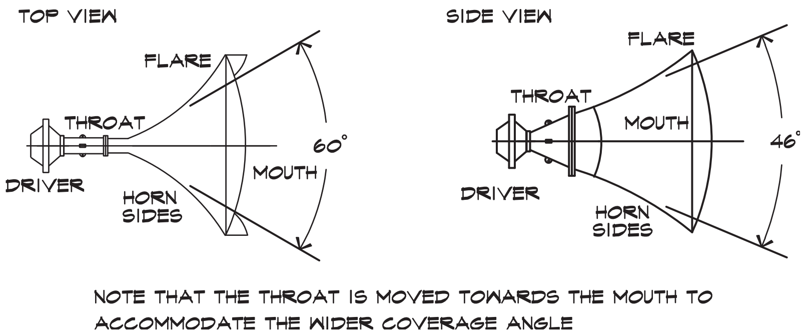
At mid frequencies, where the size of the driver mouth is small with respect to the wavelength, the sound illuminates the side walls and allows them to control the directional pattern emanating from the horn. With constant directivity horns, the side walls are either straight or slightly curved, having different centers of expansion in the horizontal and vertical dimensions. This innovation was introduced by Paul Klipsch in 1951 and has been used in most subsequent horn designs. It allows for a different coverage angle in the horizontal and vertical planes.

Constant-Directivity Horns

Constant-directivity horns are specifically designed to provide an even frequency distribution with direction. A typical example is shown in Fig. 6.28. In the ideal case, the sound spectrum measured at any particular location should be no different from that measured at any other location within the field of the horn's coverage. One result of this type of behavior is that the spectrum at any point is quite close to the actual power spectrum of the driver, since the power is evenly distributed. This feature is critical for successful sound system design and greatly simplifies the design process.

Modern constant directivity horn design started with the work of D. B. (Don) Keele, Jr., John Gilliom, and Ray Newman at Electrovoice in the middle of the 1970s. Their work introduced the features that are the central methods for controlling directivity in all current horn design. Their first design idea was a contraction in the throat immediately following

FIGURE 6.28 Constant Directivity Horn—Nominal 60 × 40 (JBL 2365)



the driver. This detail is emphasized in the White horn series they designed; however, the idea predates this horn (Keele, 1983). The narrow throat allowed the use of a larger driver and improved the directional control by letting the sound illuminate the sides of the horn, without the high-frequency beaming usually associated with larger throat sizes.

The second feature was a conical-exponential (CE) throat shape, which consisted of an exponential throat over a certain distance, after which there was a smooth transition into a conical (straight-sided) shape. The combination of these two curves allowed the control of low-frequency impedance by the use of the quasi-exponential throat expansion, and still maintained excellent directional coverage afforded by the conical shape of the sides.

The third step the group took was to address the problem of mid-range narrowing, present in most horns before this design. Their approach was to flare the mouth of the horn at a point, which was about two-thirds the distance from the beginning of the conical section to the mouth. The flare was added at an angle, which was about twice the angle of the original conical section. The added flare resulted in the high frequencies seeing one mouth size and the lower frequencies seeing another. The flare also allowed the transition between the horn mouth and the surrounding air to be less abrupt. The sound pressure distribution across the mouth of the horn was no longer constant but was higher in the center of the horn. The horn mouth no longer looked like a pure piston in a baffle and as a result the mid-frequency narrowing problems (predicted by the piston model) that were associated with previous designs were no longer present.

The White horn series was highly successful. The design gave good horizontal-directivity control without the mid-frequency beaming that had been associated with most previous horns. The vertical frequency control was not emphasized in the design in favor of a smaller vertical dimension. This shows the design tradeoff that is made between horn mouth height and the capability of controlling the vertical directivity over a wide range of frequencies. Because the vertical dimension of the mouth is relatively small, the frequency at which the vertical control begins is rather high—1.2 kHz. The White horn series was the first commercial product to be a true constant-directivity type.

The next chronological development in horn design was the introduction of the Mantaray horn series by Mark Ureda and Cliff Henricksen at Altec. They wanted to produce a horn with strong directivity control both horizontally and vertically. Ureda and Henricksen decided that if bidirectional control was to be achieved, then the mouth had to be square and relatively large. Once the low-frequency limit was known, the mouth size could be determined from the piston in a baffle formula. With the mouth size fixed, the coverage angles allowed the sides to be drawn back to the driver's mouth. The large vertical mouth dimension resulted in narrow angled top and bottom walls extending back to the throat opening, which controlled length of the horn as well as the loading on the driver. In the horizontal plane the wider coverage angle resulted in the sides converging to a point that was displaced further down the horn from the throat. The throat was connected to this point by an opening having a relatively narrow cross section. The Mantaray design used the flare idea developed by Keele et al., but its flare started further down the horn than the two-third's point.

This horn had some advantages over previous designs. Because the vertical mouth dimension was large, the narrowest side wall angle expanded over a longer distance and was connected directly to the driver. This displaced the driver from the throat of the wide-angle portion of the horn and made it easier for the driver to couple to the horn. This feature is an advantage particularly when using 2-inch drivers, which have a difficult time driving directly into a 90° angle opening without beaming.

The Mantaray horns emphasize the control of directivity in both planes, but do so at the expense of low-frequency loading, which is brought about by an exponential area expansion. The sides of the Mantaray are virtually straight in both the horizontal and vertical planes. The Altec patent claims that the straight-sided walls improve the waistbanding effect (a sideways lobing of the midfrequencies), which is said to have been found in other designs. The Mantaray design yields a directivity that is highly controlled down to 800 Hz and out to 20 kHz. Because of their size, they require more space than other horns. The tradeoffs are good vertical directivity control and excellent high-frequency response against a large physical size and minimal low-frequency loading.

Another horn manufacturer, JBL, subsequently developed its own constant directivity horns, which were also designed by Keele. In these designs, called Bi-Radial horns, Keele used a very general polynomial formula to develop the side shape from the throat to the mouth.

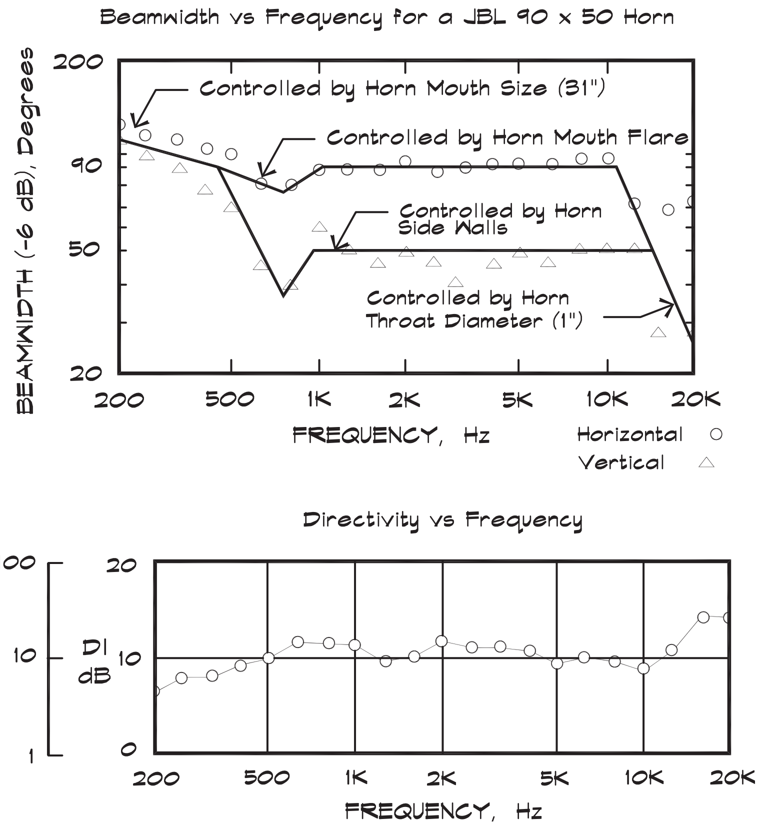
$$y = a + bx + cx^n \tag{6.72}$$

where x = distance along the centerline from the mouth
y = distance perpendicular to the x axis
a = half the throat height
$b = [\tan(0.9\,\text{beamwidth})]/2$
$c = [w/2 - bL - a]/L^n$
n = constant between 2 and 8
L = horn length
w = mouth width

The Bi-Radial design, as with all horn designs, is a compromise. The mouth size is selected from the piston in a baffle equation. The throat expansion is quasi-exponential as it is a combination of constant linear taper and an exponential area expansion. The loading is better than the straight-sided Mantaray conical loading, but not as good as the more purely exponential loading used in the Electrovoice designs. The mouth size is large in both dimensions, so that the directivity is controlled to relatively low frequencies. Probably the most interesting design feature is the use of the equation for the flare rate. According to Keele (1983), this curve gives a smooth response both along the horizontal and vertical directions as well as off-axis between the two planes. The line determined by the equation is rotated about a point on two sides and hence the name Bi-Radial. The horn does not have a contraction near the throat so that the horn tends to beam above 10 kHz. The Bi-Radial design is generally a good compromise between loading and directivity. It provides control in both directions and smooth off-axis response.

In Fig 6.29, we can see the various regions of the horn and what controls the directivity in each region. At low frequencies, the size of the mouth is the determinant. It sets the frequency at which the horn begins to control. Above the control point, the angle of the horn sides sets the coverage pattern. At very high frequencies, the diameter of the driver opening controls the beamwidth since the sound no longer interacts with the sides of the horn. Between the side-angle region of control and the low-frequency cutoff point, there is waistbanding or narrowing of the coverage pattern. This is controlled by the horn flare, which prevents the mouth of the horn from acting like a pure piston in a baffle.

FIGURE 6.29 Beamwidth and Directivity of Constant Directivity Horns (Long, 1983)



Cabinet Arrays

In recent years, it has become popular for manufacturers to provide two- or three-way cabinets that are trapezoidal-shaped. The idea behind the shape is that the cabinets may be arrayed side by side to provide the appropriate coverage. With the advent of packaged computer design programs, the directivity patterns of these products often are buried in the computer code and not available to the designer. Cabinets are subject to the same physical limitations on directional control imposed by the size of the radiating components as any other device. Arraying cabinets in a line can does not narrow the coverage angle in the plane normal to the line, where directional control may also be needed. At frequencies below the point where the spacing is equal to a wavelength, horizontal stacking will narrow the coverage angle in the horizontal plane, which may or may not be useful. Stacking cabinets in a line can control directivity over a certain frequency range, where the components act as a line array, but is probably not particularly beneficial above that frequency.

Baffled Low-Frequency Systems

The installation of low-frequency cabinets in a baffle wall is a technique that can be used to increase their directivity somewhat. The theoretical result is illustrated in Fig. 6.21. The materials used in constructing these baffles are usually not 100% reflective at the frequencies of interest. For example, a single sheet of drywall is about 30% absorptive at 125 Hz and as much as 40% absorptive at slightly lower frequencies. This significantly reduces the theoretical effectiveness of a lightweight baffle wall. Double drywall, being about 20% absorptive at 125 Hz and 35% absorptive at 80 Hz, is not significantly better.

Since subwoofer cabinets usually are used below 125 Hz, where the wavelength is about 9 ft (2.8 m) long, it can be helpful to place them on the floor or up against a concrete wall where they would be less than a third of a wavelength away from their image source. From Fig. 6.9 we can see that a power doubling, if not a pressure doubling, would probably be achieved for $k\ d = 2$. When there are two reflecting surfaces, the floor and the wall, a 6 dB increase could be achieved.

Baffle walls, if improperly constructed, can sometimes do more harm than good. When an unsealed gap is left around a baffled loudspeaker, it can become the throat of a Helmholtz resonator with the enclosed volume behind the baffle wall acting as the resonator volume. The resulting resonance can significantly color the sound and offset the advantage of a small increase in level at low frequencies provided by the wall.