LECTURE 8

PROGRAM EVALUATION AND REVIEW TECHNIQUE (PERT)

OUTLINE

- REVIEW MID TEST
- INTRODUCTION
- DEFINITION
- CONCEPT OF PERT
- PROJECT Y
- PERT CALCULATION
- ESTIMATING THE PROBABILITY OF COMPLETION DATES
- EXERCISE

REVIEW MID TEST

INTRODUCTION

CPM and PERT are:

- Graphically display the precedence relationships & sequence of activities
- Estimate the project's duration
- Identify critical activities that cannot be delayed without delaying the project
- Estimate the amount of slack associated with non-critical activities

DEFINITION

(PERT) is an event-oriented NETWORK ANALYSIS TECHNIQUE used to estimate project duration WHEN individual ACTIVITY DURATION estimates are highly UNCERTAIN.

PERT" developed by the United States Department of Defense as a management tool with an acronym "Program Evaluation and Review Technique".

Background

In the 1950s, the U.S. Navy had a budget overrun and a schedule delay of as much as 50% in the Polaris missile system project.

The main problem was the lack of any relevant historical data. The project team—the U.S. Navy (the owner), Lockheed Aircraft Corporation (the prime contractor), and Booz Allen Hamilton (the management consultant team)—launched a joint research effort to develop a tool to assist in the planning of the Polaris project.

The objective was to devise a method that predicts the completion date of a project with a certain likelihood using the theory of probability.

In 1958, this tool was developed under the name program evaluation and review technique and later became known by its acronym, PERT. The PERT system was adopted in its early days by the Department of Defense (DoD) under the name program evaluation procedure (PEP).

CONCEPT OF PERT

- PERT uses a **probabilistic time estimates**

- PERT requires the user to SET THREE DURATIONS that constitute the practical range of the duration for each activity

Estimating assumptions on Set Duration

- An estimate is a qualified guess.
- Every estimate is based on assumptions.
- These estimate assumptions need to be specified so that the basis of the estimate is known and validity of the assumptions can be assessed.
- For example, a work effort estimate is usually based on a level of skill and expertise. (If a lower level of skill and expertise is utilized then it is likely that the task will require more effort.)

CONCEPT OF PERT

Formula

- To: Optimistic Duration

The optimistic duration is the amount of time the activity will take if everything goes smoothly and efficiently

- Tm: Most Likely Duration

The Most Likely duration is the duration under normal scenario

- Tp: Pessimistic Duration

The pessimistic duration is the duration under the worst-case scenario

PERT weighted average =
optimistic time + 4X most likely time + pessimistic time

CONCEPT OF PERT

Example:

Assembling and Erecting the False work for an Elevated Slab will most likely require 8 days. If all goes well, without interruption, the duration may be cut to 10 days. However, in the practically worst-case scenario, this activity may take 24 days.

The preceding values are estimated by the scheduler or project manager, who uses his or her EXPERIENCE AND GOOD JUDGMENT to do so

CONCEPT OF PERT

Example:

where optimistic time= 8 days,
most likely time = **10 days**, and
pessimistic time = 24 days

PERT weighted average =

$$\frac{8 \text{ workdays} + 4 \times 10 \text{ workdays} + 24 \text{ workdays}}{6} = \textbf{12 days}$$

Therefore, you'd use **12 days** on the network diagram instead of 10

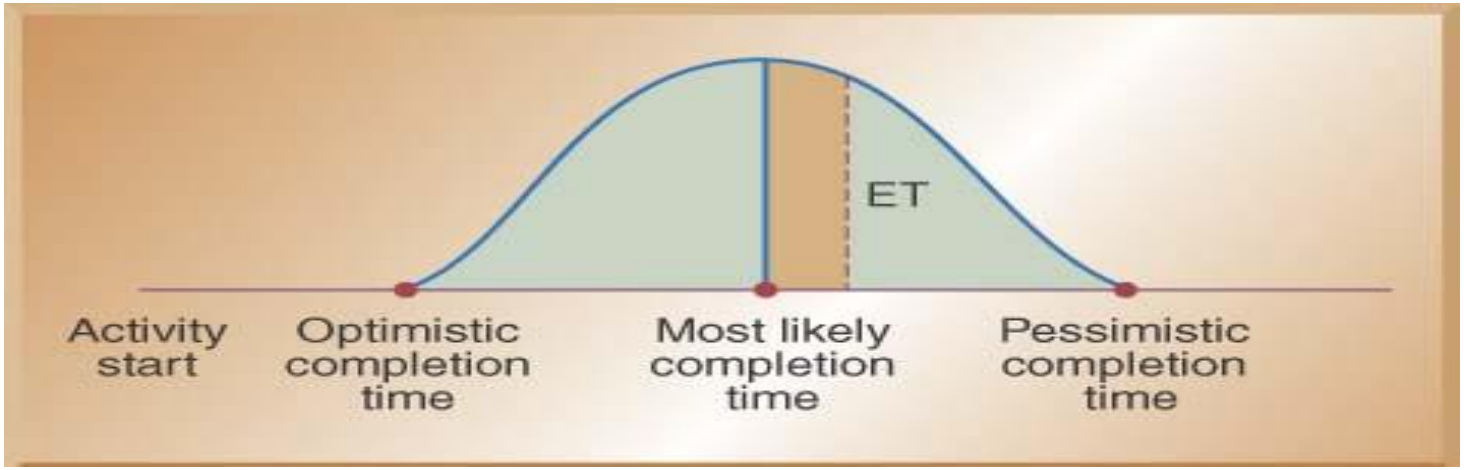
when using PERT for the above example

Project Y

Activity	Description	Optimistic time	Most likely time	Pessimistic time
A	Develop product specifications	2	4	6
B	Design manufacturing process	3	7	10
C	Source & purchase materials	2	3	5
D	Source & purchase tooling & equipment	4	7	9
E	Receive & install tooling & equipment	12	16	20
F	Receive materials	2	5	8
G	Pilot production run	2	2	2
H	Evaluate product design	2	3	4
I	Evaluate process performance	2	3	5
J	Write documentation report	2	4	6
K	Transition to manufacturing	2	2	2

PERT CALCULATION

- **A typical beta distribution is shown below, note that it has definite end points**
- **The EXPECTED TIME (ET) for finishing each activity is a weighted average**



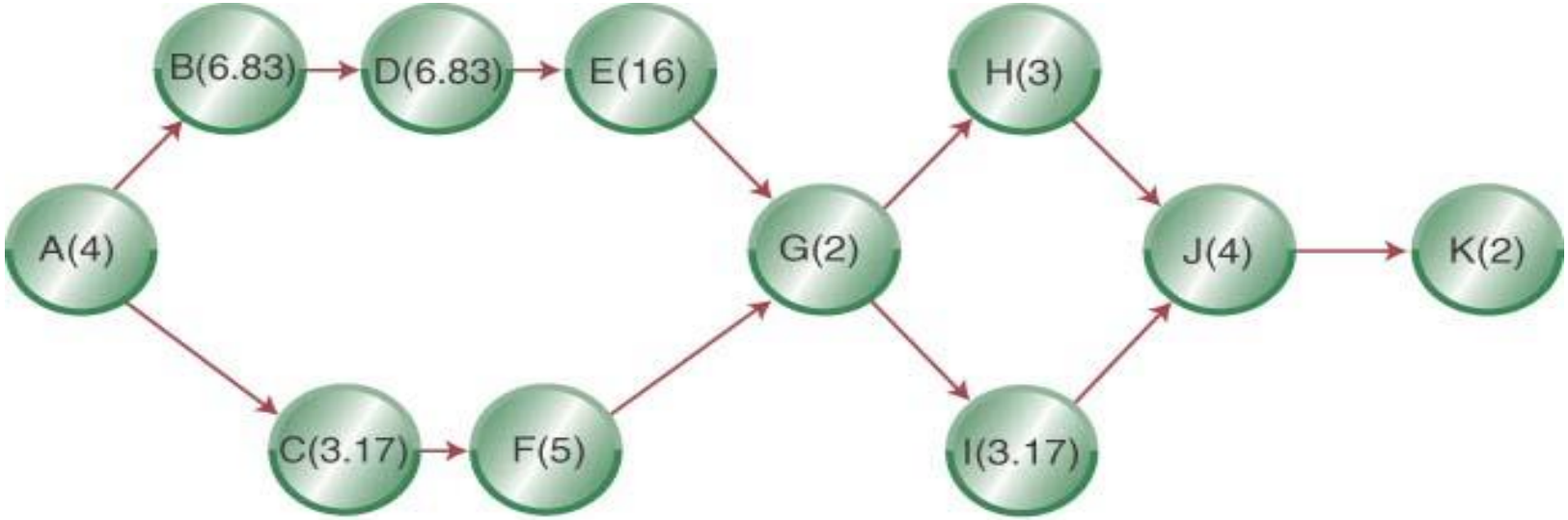
$$\text{Exp. time} = \frac{\text{optimistic} + 4(\text{most likely}) + \text{pessimistic}}{6}$$

Calculating Expected Task Times

$$\text{Expected time} = \frac{\text{optimistic} + 4(\text{most likely}) + \text{pessimistic}}{6}$$

Activity	Optimistic time	Most likely time	Pessimistic time	Expected time
A	2	4	6	4
B	3	7	10	6.83
C	2	3	5	3.17
D	4	7	9	6.83
E	12	16	20	16
F	2	5	8	5
G	2	2	2	2
H	2	3	4	3
I	2	3	5	3.17
J	2	4	6	4
K	2	2	2	2

Network Diagram with Expected Activity Times



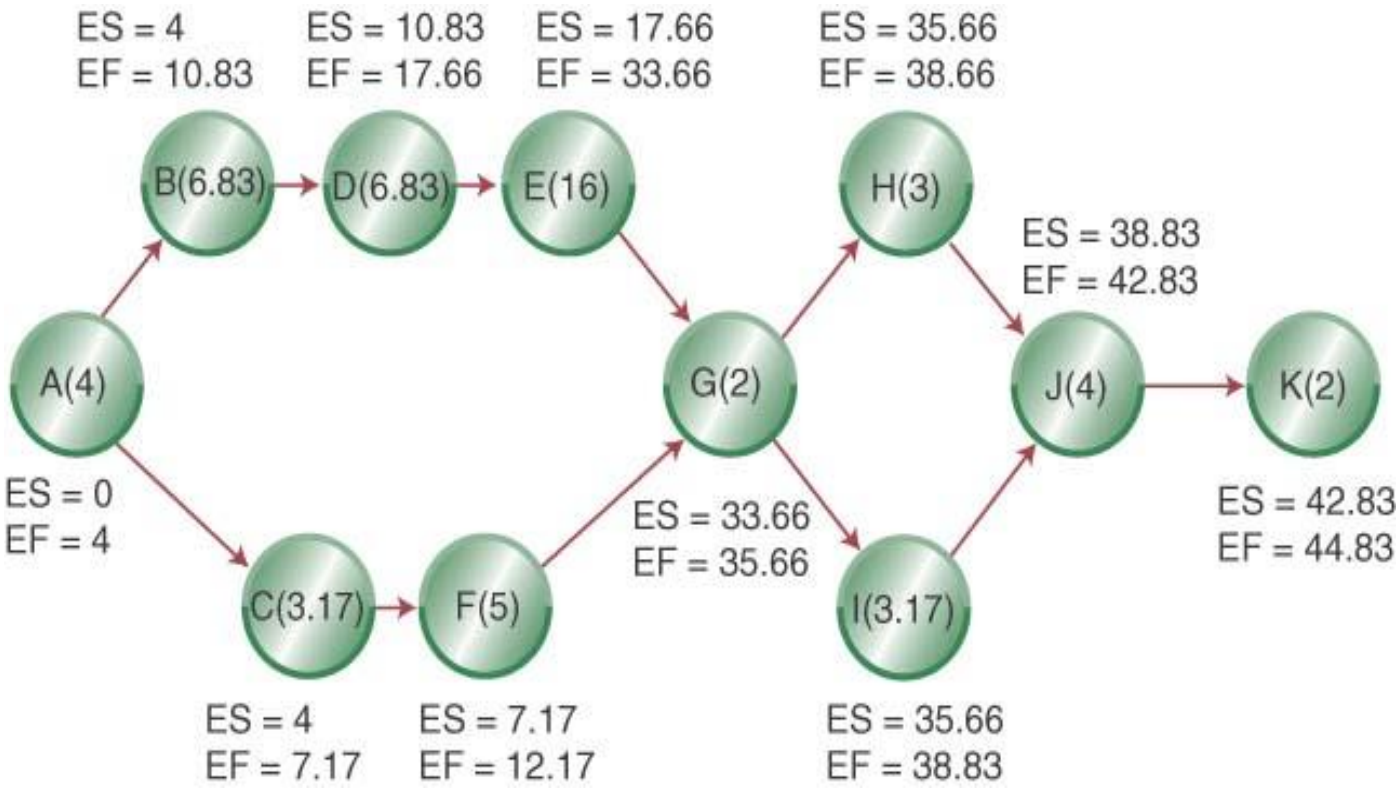
Connected paths
1. A, B, D, E, G, H, J, K
2. A, B, D, E, G, I, J, K
3. A, C, F, G, H, J, K
4. A, C, F, G, I, J, K

Estimated Path Durations through the Network

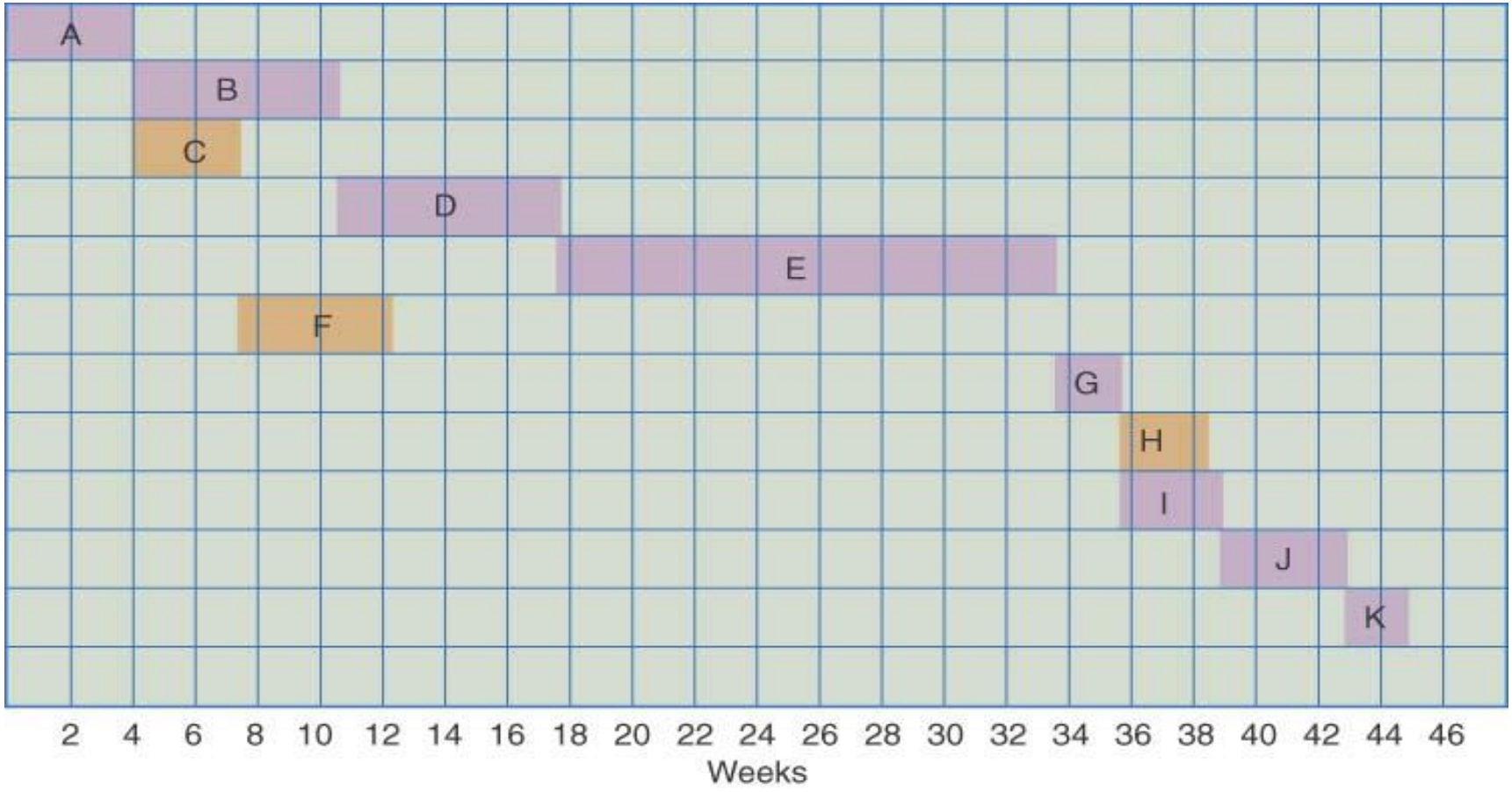
Activities on paths	Expected duration
ABDEGHJK	44.66
ABDEGIJK	44.83
ACFGHJK	23.17
ACFGIJK	23.34

- **ABDEGIJK** is the expected critical path & the project has an expected duration of **44.83 weeks**

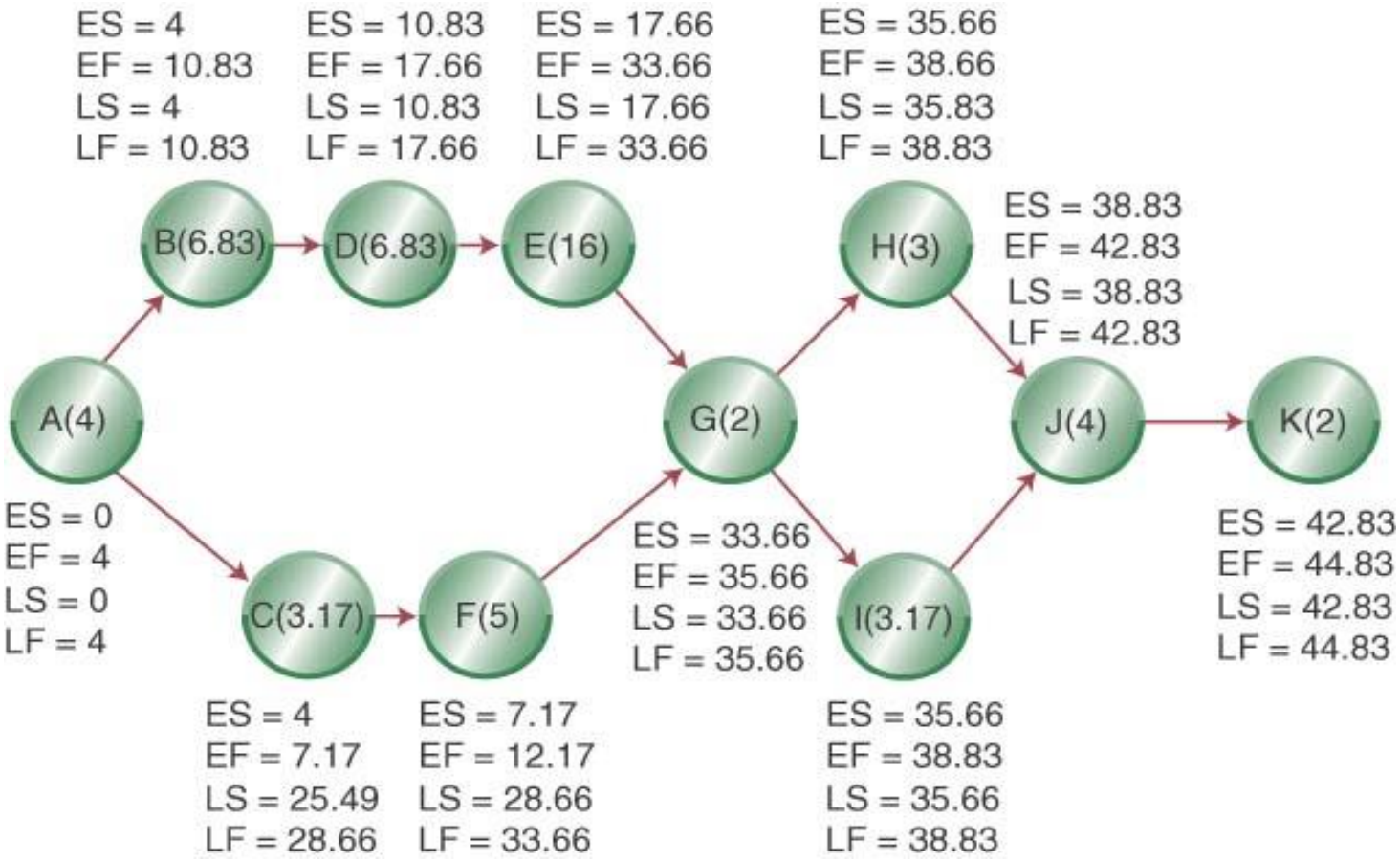
Adding ES and EF to Network



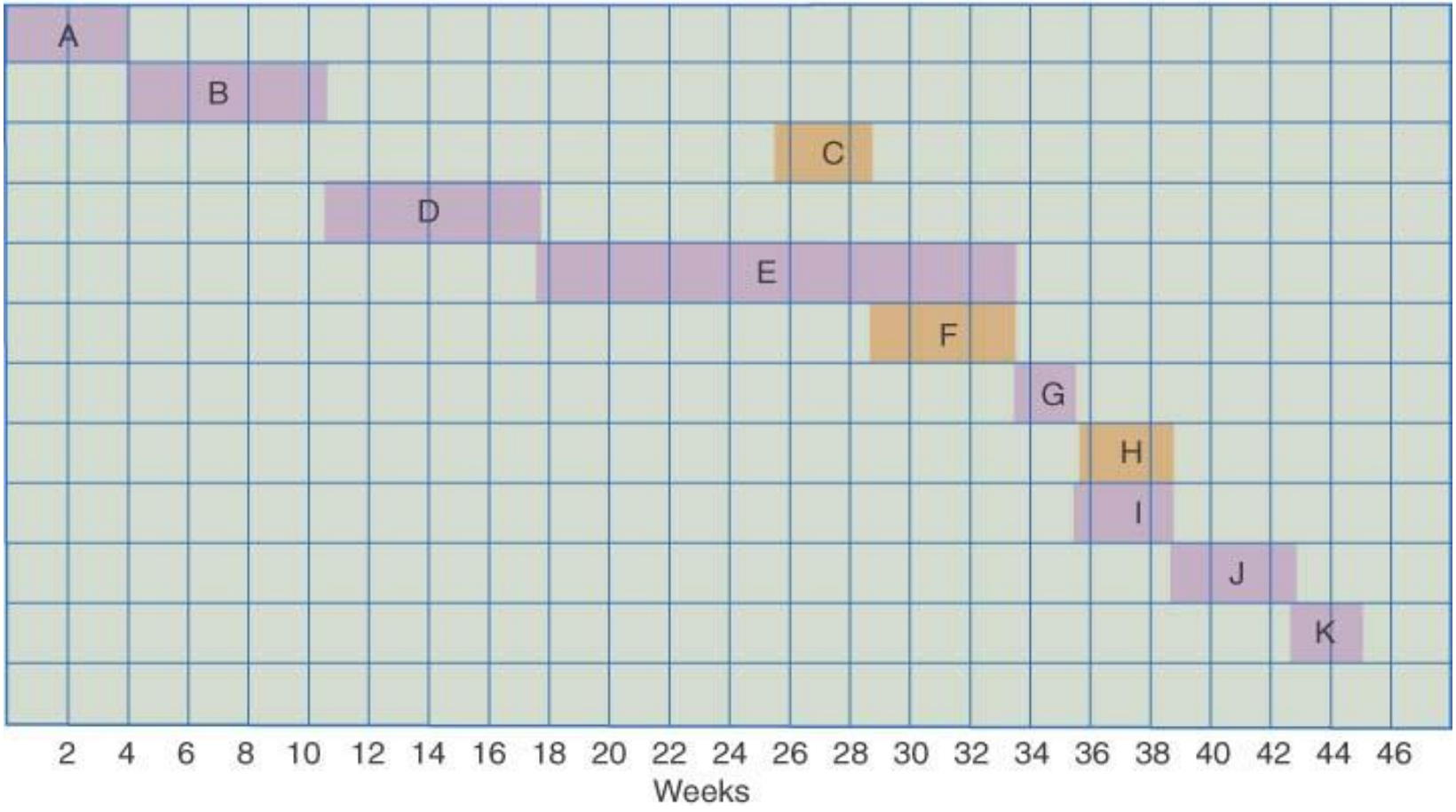
Gantt Chart Showing Each Activity Finished at the Earliest Possible Start Date



Adding LS and LF to Network



Gantt Chart Showing the Latest Possible Start Times if the Project Is to Be Completed in 44.83 Weeks



Estimating the Probability of Completion Dates

- Standard deviation and variance
  - Standard deviation (SD) is the average deviation from the estimated time
    - SD=(T_P-T_0)/6 {read as (pessimistic-optimistic)/6}
    - As a general rule, the higher the standard deviation the greater the amount of uncertainty
  - Variance (V) reflects the spread of a value over a normal distribution
    - $V=SD^2$ (Standard deviation squared) or $\sigma^2 = \left(\frac{\mathrm{Tp} - To}{6} \right)^2$

Table 11.1 Z Table: Cumulative Probability of the Standard Normal Distribution

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990

Project Activity Variance

Activity	Optimistic	Most Likely	Pessimistic	Variance
A	2	4	6	0.44
B	3	7	10	1.36
C	2	3	5	0.25
D	4	7	9	0.69
E	12	16	20	1.78
F	2	5	8	1.00
G	2	2	2	0.00
H	2	3	4	0.11
I	2	3	5	0.25
J	2	4	6	0.44
K	2	2	2	0.00

Variances of Each Path through the Network

Path Number	Activities on Path	Path Variance (weeks)
1	A,B,D,E,G,H,J,k	4.82
2	A,B,D,E,G,I,J,K	4.96
3	A,C,F,G,H,J,K	2.24
4	A,C,F,G,I,J,K	2.38

PROBABILITY OF COMPLETION

Assume Calculating the probability of finishing the project in 48 weeks

- Use the z values in Appendix B to determine probabilities
- e.g. probability for path 1 is $$z = \left(\frac{48 \text{ weeks} - 44.66 \text{ weeks}}{\sqrt{4.82}} \right) = 1.52$$

Path Number	Activities on Path	Path Variance (weeks)		z-value	Probability of Completion
1	A,B,D,E,G,H,J,k	4.82	44.66	1.5216	0.9357
2	A,B,D,E,G,I,J,K	4.96	44.83	1.4215	0.9222
3	A,C,F,G,H,J,K	2.24	23.17	16.5898	1.000
4	A,C,F,G,I,J,K	2.38	23.34	15.9847	1.000

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2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990

Exercise

In a CPM network, the critical path includes five activities. Their durations are tabulated next.

	Duration (Days)		
Activity	Optimistic (T_o)	Most Likely (T_m)	Pessimistic (T_p)
A	2	4	7
B	5	8	14
C	4	6	8
D	2	2	2
E	7	10	21

Compute the following nine values:

With a Probability of Finishing Project is **Total Amount of Expected Duration**

1. The probability that the project will finish by the end of day 32
2. The probability that the project will finish by the end of day 34

Exercise

Expected Time: $\frac{\text{optimistic time + 4X most likely time + pessimistic time}}{6}$

- Standard deviation and variance
  - $SD=(T_P-T_0)/6$ {read as (pessimistic-optimistic)/6}

  – Variance (V) reflects the spread of a value over a normal distribution
    - $V=SD^2$ (Standard deviation squared)

THANK YOU