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Mata Kuliah : Dinamika Struktur & Pengantar Rekayasa Kegempaan
Kode : TSP – 302
SKS : 3 SKS

Single Degree of Freedom System

Numerical Evaluation of Dynamic Response

Pertemuan - 5

- TIU :
  - Mahasiswa dapat menjelaskan tentang teori dinamika struktur.
  - Mahasiswa dapat membuat model matematik dari masalah teknis yang ada serta mencari solusinya.

- TIK :
  - Mahasiswa mampu menghitung respon sistem struktur akibat beban dinamis melalui metode numerik

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- Sub Pokok Bahasan :
  - Interpolasi
  - Central Difference
  - Newmark Method

Numerical Evaluation of Dynamic Response

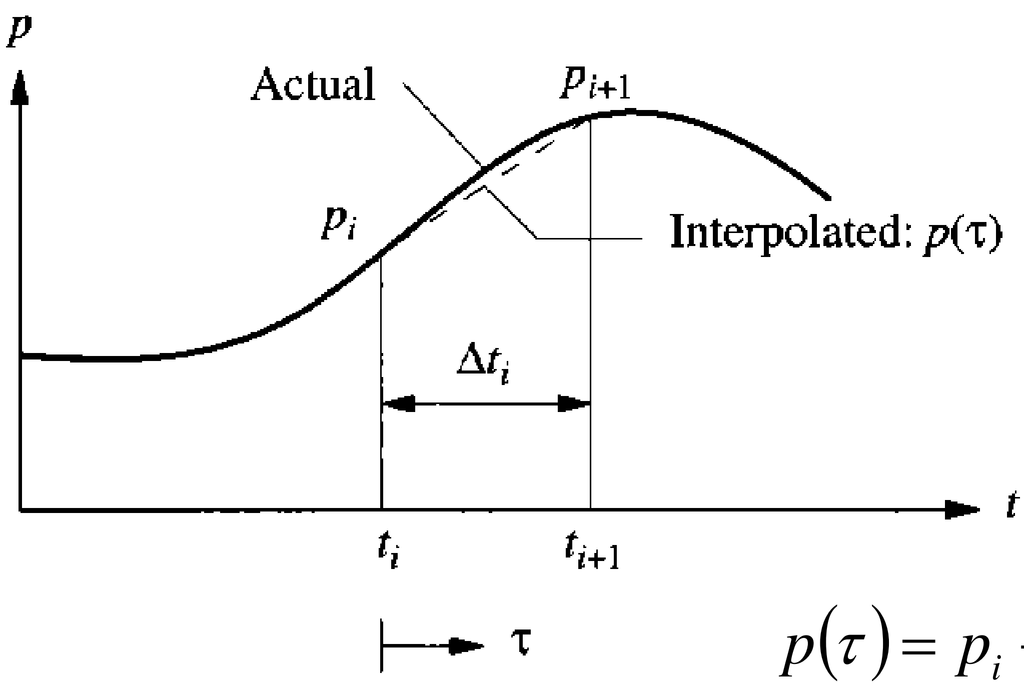
- Analytical solution of the equation for a SDOF System is usually *not possible* if the excitation – applied force p(t) or ground acceleration – varies arbitrarily with time or if the system is nonlinear
- Methods of Numerical evaluation are :

  - interpolation of excitation

  - central difference

  - Newmark method (Linear Acceleration & Average Acc.)

Interpolation of Excitation



$$p(\tau) = p_i + \frac{\Delta p_i}{\Delta t_i}\tau$$

$$\Delta p_i = p_{i+1} - p_i$$

Interpolation of Excitation

TABLE 5.2.1 COEFFICIENTS IN RECURRENCE FORMULAS ($\zeta < 1$)

$$A = e^{-\zeta\omega_n \Delta t}\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_D \Delta t + \cos\omega_D \Delta t\right)$$

$$B = e^{-\zeta\omega_n \Delta t}\left(\frac{1}{\omega_D}\sin\omega_D \Delta t\right)$$

$$C = \frac{1}{k}\left\{\frac{2\zeta}{\omega_n \Delta t} + e^{-\zeta\omega_n \Delta t}\left[\left(\frac{1-2\zeta^2}{\omega_D \Delta t} - \frac{\zeta}{\sqrt{1-\zeta^2}}\right)\sin\omega_D \Delta t - \left(1 + \frac{2\zeta}{\omega_n \Delta t}\right)\cos\omega_D \Delta t\right]\right\}$$

$$D = \frac{1}{k}\left[1 - \frac{2\zeta}{\omega_n \Delta t} + e^{-\zeta\omega_n \Delta t}\left(\frac{2\zeta^2 - 1}{\omega_D \Delta t}\sin\omega_D \Delta t + \frac{2\zeta}{\omega_n \Delta t}\,\mathrm{cc}\ldots\right)\right]$$

$$A' = -e^{-\zeta\omega_n \Delta t}\left(\frac{\omega_n}{\sqrt{1-\zeta^2}}\sin\omega_D \Delta t\right)$$

$$B' = e^{-\zeta\omega_n \Delta t}\left(\cos\omega_D \Delta t - \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_D \Delta t\right)$$

$$C' = \frac{1}{k}\left\{-\frac{1}{\Delta t} + e^{-\zeta\omega_n \Delta t}\left[\left(\frac{\omega_n}{\sqrt{1-\zeta^2}} + \frac{\zeta}{\Delta t\sqrt{1-\zeta^2}}\right)\sin\omega_D \Delta t + \frac{1}{\Delta t}\cos\omega_D \Delta t\right]\right\}$$

$$D' = \frac{1}{k\,\Delta t}\left[1 - e^{-\zeta\omega_n \Delta t}\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_D \Delta t + \cos\omega_D \Delta t\right)\right]$$

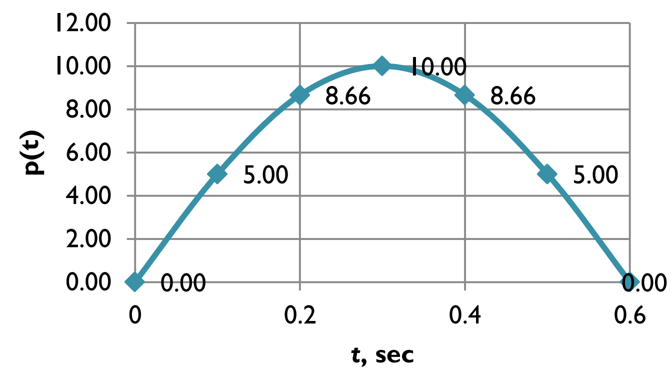
$$u_{i+1} = Au_i + B\dot{u}_i + Cp_i + Dp_{i+1}$$

$$\dot{u}_{i+1} = A'u_i + B'\dot{u}_i + C'p_i + D'p_{i+1}$$

Interpolation of Excitation (Example 1)

An SDF system has the following properties :

- m = 4.500 kg-sec^2/m
- k = 178.400 kgf/m
- T_n = 1 sec (ω_n = 6,283 rad/sec)
- ξ = 0,05.
- Determine the response u(t) of this system to p(t) = 4,500 sin(πt/0,6) kgf, defined by the half-cycle sine pulse by using piecewise linear interpolation of p(t) with Δt = 0,1 sec.



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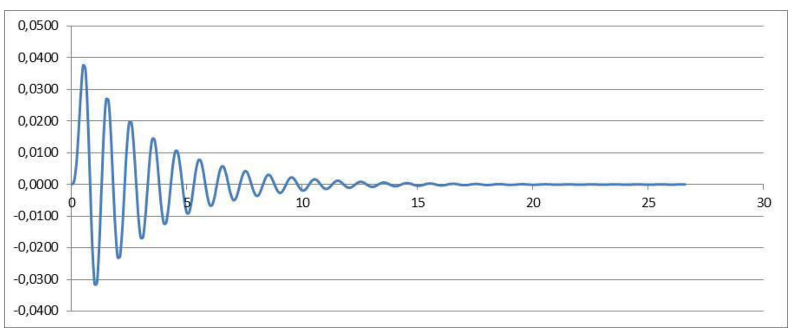
t_i	p_i	$C.p_i$	$D.p_{i+1}$	$\dot{u}_i$	$B.\dot{u}_i$	u_i	$A.u_i$	$C'.p_i$	$D'.p_{i+1}$	$A'.u_i$	$B'.\dot{u}_i$
0	0,00	0,0000	0,0008	0,0000	0,0000	0,0000	0,0000	0,0000	0,0237	0,0000	0,0000
0,1	2250,00	0,0016	0,0014	0,0237	0,0021	0,0008	0,0007	0,0216	0,0410	-0,0029	0,0179
0,2	3897,11	0,0027	0,0016	0,0777	0,0070	0,0058	0,0047	0,0375	0,0474	-0,0207	0,0586
0,3	4500,00	0,0031	0,0014	0,1228	0,0111	0,0160	0,0130	0,0433	0,0410	-0,0576	0,0927
0,4	3897,11	0,0027	0,0008	0,1194	0,0108	0,0287	0,0233	0,0375	0,0237	-0,1030	0,0902
0,5	2250,00	0,0016	0,0000	0,0483	0,0044	0,0376	0,0306	0,0216	0,0000	-0,1352	0,0365
0,6	0,00	0,0000	0,0000	-0,0771	-0,0070	0,0365	0,0296	0,0000	0,0000	-0,1312	-0,0582
0,7	0,00	0,0000	0,0000	-0,1894	-0,0172	0,0227	0,0184	0,0000	0,0000	-0,0814	-0,1430
0,8	0,00	0,0000	0,0000	-0,2244	-0,0203	0,0012	0,0010	0,0000	0,0000	-0,0044	-0,1694
0,9	0,00	0,0000	0,0000	-0,1739	-0,0158	-0,0193	-0,0157	0,0000	0,0000	0,0695	-0,1313
1	0,00	0,0000	0,0000	-0,0618	-0,0056	-0,0315	-0,0256	0,0000	0,0000	0,1131	-0,0467



u_i in m

$$u_{i+1} = Au_i + B\dot{u}_i + Cp_i + Dp_{i+1}$$

$$\dot{u}_{i+1} = A'u_i + B'\dot{u}_i + C'p_i + D'p_{i+1}$$

Central Difference Method

TABLE 5.3.1 CENTRAL DIFFERENCE METHOD

1.0 *Initial calculations*

1.1 $\ddot{u}_0 = \dfrac{p_0 - c\dot{u}_0 - ku_0}{m}$.

1.2 $u_{-1} = u_0 - \Delta t \dot{u}_0 + \dfrac{(\Delta t)^2}{2}\ddot{u}_0$.

1.3 $\hat{k} = \dfrac{m}{(\Delta t)^2} + \dfrac{c}{2\Delta t}$.

1.4 $a = \dfrac{m}{(\Delta t)^2} - \dfrac{c}{2\Delta t}$.

1.5 $b = k - \dfrac{2m}{(\Delta t)^2}$.

2.0 *Calculations for time step i*

2.1 $\hat{p}_i = p_i - au_{i-1} - bu_i$.

2.2 $u_{i+1} = \dfrac{\hat{p}_i}{\hat{k}}$.

2.3 If required: $\dot{u}_i = \dfrac{u_{i+1} - u_{i-1}}{2\Delta t}$, $\ddot{u}_i = \dfrac{u_{i+1} - 2u_i + u_{i-1}}{(\Delta t)^2}$.

3.0 *Repetition for the next time step*

Replace i by $i + 1$ and repeat steps 2.1, 2.2, and 2.3 for the next time step.

Central Difference Method is stable if :

$$\frac{\Delta t}{T_n} < \frac{1}{\pi}$$

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Central Difference Method (Example 2)

Solve Example 1, by the central difference method using Δt = 0,1 sec.

t_i	p_i	u_{i-1}	u_i	p_i'	u_{i+1}
0	0,00	0,0000	0,0000	0,0000	0,0000
0,1	2250,00	0,0000	0,0000	2250,0000	0,0048
0,2	3897,11	0,0000	0,0048	7395,2181	0,0159
0,3	4500,00	0,0048	0,0159	13884,5060	0,0299
0,4	3897,11	0,0159	0,0299	18538,8179	0,0399
0,5	2250,00	0,0299	0,0399	18033,8488	0,0389
0,6	0,00	0,0399	0,0389	10627,9866	0,0229
0,7	0,00	0,0389	0,0229	-411,7948	-0,0009
0,8	0,00	0,0229	-0,0009	-10620,7730	-0,0229
0,9	0,00	-0,0009	-0,0229	-16125,5428	-0,0347
1	0,00	-0,0229	-0,0347	-15096,8118	-0,0325

u_i in m

Newmark's Method

TABLE 5.4.2 NEWMARK'S METHOD: LINEAR SYSTEMS

Special cases
(1) Average acceleration method ($\gamma = \frac{1}{2}$, $\beta = \frac{1}{4}$)
(2) Linear acceleration method ($\gamma = \frac{1}{2}$, $\beta = \frac{1}{6}$)

1.0 *Initial calculations*

1.1 $\ddot{u}_0 = \dfrac{p_0 - c\dot{u}_0 - ku_0}{m}$.

1.2 Select Δt.

1.3 $\hat{k} = k + \dfrac{\gamma}{\beta\,\Delta t}c + \dfrac{1}{\beta(\Delta t)^2}m$.

1.4 $a = \dfrac{1}{\beta\,\Delta t}m + \dfrac{\gamma}{\beta}c$; and $b = \dfrac{1}{2\beta}m + \Delta t\left(\dfrac{\gamma}{2\beta} - 1\right)c$.

2.0 *Calculations for each time step, i*

2.1 $\Delta\hat{p}_i = \Delta p_i + a\dot{u}_i + b\ddot{u}_i$.

2.2 $\Delta u_i = \dfrac{\Delta\hat{p}_i}{\hat{k}}$.

2.3 $\Delta\dot{u}_i = \dfrac{\gamma}{\beta\,\Delta t}\Delta u_i - \dfrac{\gamma}{\beta}\dot{u}_i + \Delta t\left(1 - \dfrac{\gamma}{2\beta}\right)\ddot{u}_i$.

2.4 $\Delta\ddot{u}_i = \dfrac{1}{\beta(\Delta t)^2}\Delta u_i - \dfrac{1}{\beta\,\Delta t}\dot{u}_i - \dfrac{1}{2\beta}\ddot{u}_i$.

2.5 $u_{i+1} = u_i + \Delta u_i$, $\dot{u}_{i+1} = \dot{u}_i + \Delta\dot{u}_i$, $\ddot{u}_{i+1} = \ddot{u}_i + \Delta\ddot{u}_i$.

3.0 *Repetition for the next time step.* Replace i by $i + 1$ and implement steps 2.1 to 2.5 for the next time step.

Newmark's method is stable if :

$$\frac{\Delta t}{T_n} \le \frac{1}{\pi\sqrt{2}}\frac{1}{\sqrt{\gamma - 2\beta}}$$

For γ = ½ & β = ¼

$$\frac{\Delta t}{T_n} \le \infty$$

For γ = ½ & β = 1/6

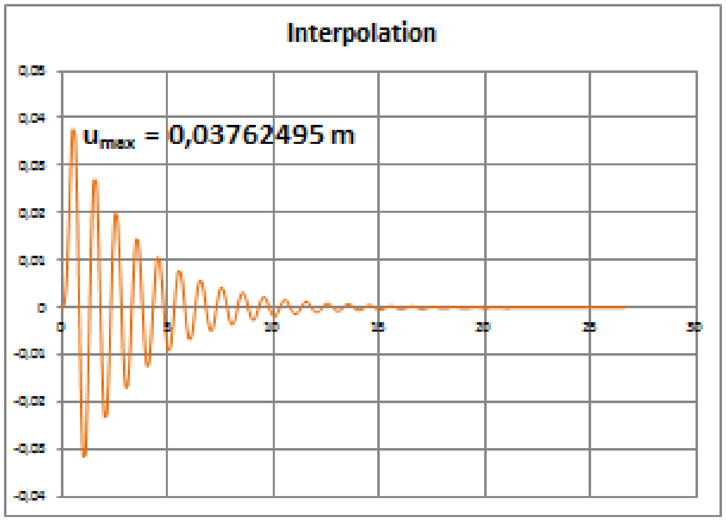
$$\frac{\Delta t}{T_n} \le 0{,}551$$

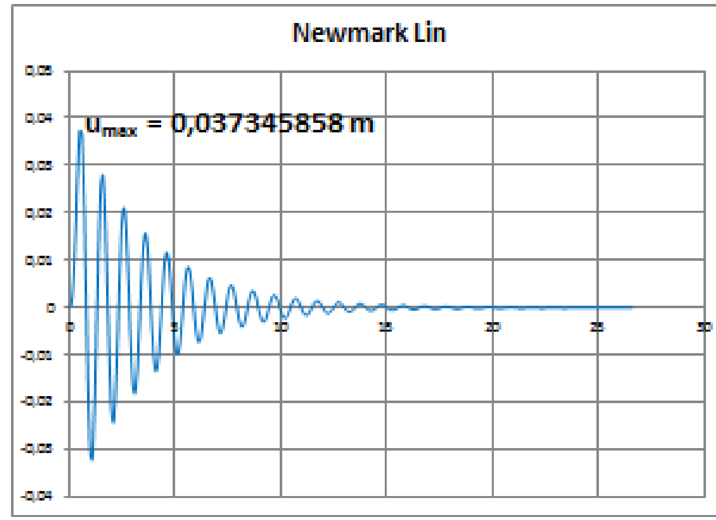
Newmark's Method – Average Acceleration (Example 3)

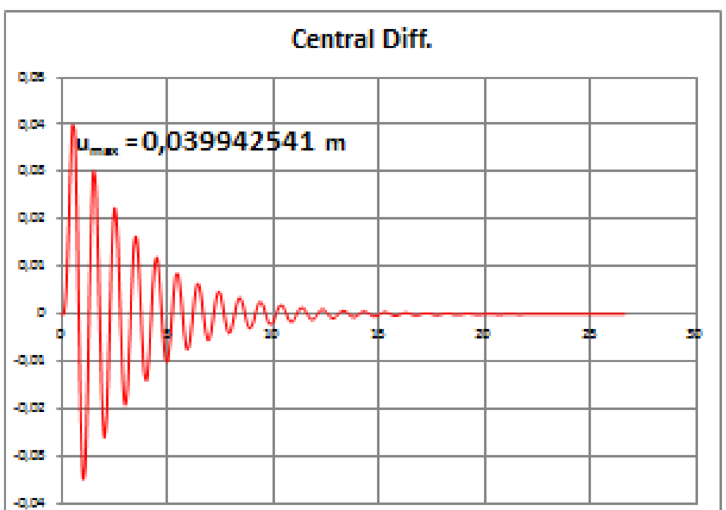
t_i	p_i	$\ddot{u}_i$	Δp_i	$\Delta p_i'$	Δu_i	$\Delta \mathring{u}_i$	$\Delta \ddot{u}_i$	$\mathring{u}_i$	u_i
0,0	0,0000	0,0000	2250,0000	2250,0000	0,0011	0,0221	0,4423	0,0000	0,0000
0,1	2250,00	0,4423	1647,1143	9733,0524	0,0048	0,0514	0,1441	0,0221	0,0011
0,2	3897,11	0,5864	602,8857	19534,2187	0,0096	0,0449	-0,2748	0,0735	0,0059
0,3	4500,00	0,3116	-602,8857	24190,5526	0,0119	0,0009	-0,6058	0,1184	0,0155
0,4	3897,11	-0,2942	-1647,1143	17855,1439	0,0088	-0,0631	-0,6742	0,1193	0,0274
0,5	2250,00	-0,9684	-2250,0000	-536,5761	-0,0003	-0,1176	-0,4156	0,0562	0,0361
0,6	0,0000	-1,3841	0,0000	-23865,0826	-0,0117	-0,1117	0,5351	-0,0614	0,0359
0,7	0,0000	-0,8490	0,0000	-39778,2274	-0,0195	-0,0447	0,8031	-0,1731	0,0242
0,8	0,0000	-0,0459	0,0000	-40857,8396	-0,0201	0,0341	0,7745	-0,2178	0,0046
0,9	0,0000	0,7286	0,0000	-27549,7397	-0,0135	0,0967	0,4760	-0,1837	-0,0155
1,0	0,0000	1,2046	0,0000	-5320,3417	-0,0026	0,1218	0,0271	-0,0871	-0,0290

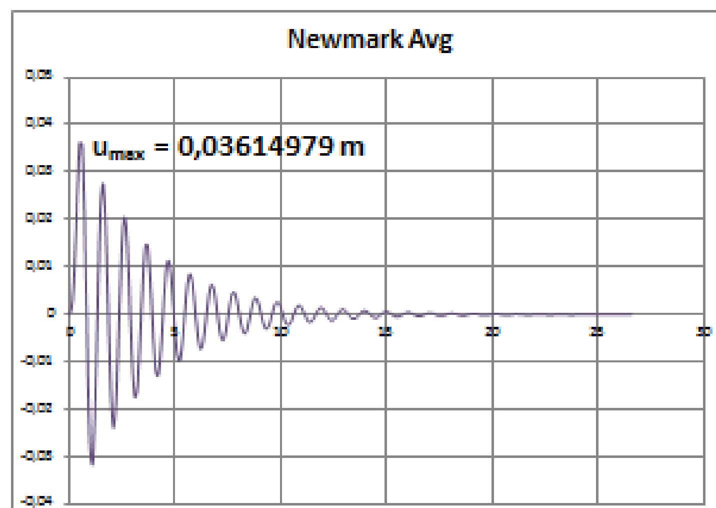
Newmark's Method – Linear Acceleration (Example 4)

t_i	p_i	$\ddot{u}_i$	Δp_i	$\Delta p_i'$	Δu_i	$\Delta \dot{u}_i$	$\Delta \ddot{u}_i$	$\dot{u}_i$	u_i
0,0	0,0000	0,0000	2250,0000	2250,0000	0,0008	0,0228	0,4556	0,0000	0,0000
0,1	2250,00	0,4556	1647,1143	14205,5370	0,0048	0,0527	0,1429	0,0228	0,0008
0,2	3897,11	0,5984	602,8857	29786,5145	0,0101	0,0452	-0,2929	0,0755	0,0056
0,3	4500,00	0,3055	-602,8857	37171,5127	0,0125	-0,0010	-0,6307	0,1207	0,0156
0,4	3897,11	-0,3252	-1647,1143	27250,5424	0,0092	-0,0669	-0,6885	0,1197	0,0281
0,5	2250,00	-1,0137	-2250,0000	-1387,4573	-0,0005	-0,1216	-0,4050	0,0528	0,0373
0,6	0,0000	-1,4187	0,0000	-38531,8427	-0,0130	-0,1126	0,5862	-0,0689	0,0369
0,7	0,0000	-0,8325	0,0000	-61880,9015	-0,0209	-0,0406	0,8534	-0,1814	0,0239
0,8	0,0000	0,0209	0,0000	-61540,0822	-0,0208	0,0419	0,7970	-0,2220	0,0030
0,9	0,0000	0,8179	0,0000	-38988,7559	-0,0132	0,1046	0,4559	-0,1801	-0,0178
1,0	0,0000	1,2738	0,0000	-3644,6624	-0,0012	0,1259	-0,0303	-0,0755	-0,0309


Interpolation
u max = 0,03762495 m


Newmark Lin
u max = 0,037345858 m


Central Diff.
u max = 0,039942541 m


Newmark Avg
u max = 0,03614979 m

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Assignment

An SDF system has the following properties :

- $m = 4.500$ kg-sec^2/m
- $k = 178.400$ kgf/m
- $T_n = 1$ sec ($\omega_n = 6,283$ rad/sec)
- $\xi = 0,05$.

Determine the response u(t) of this system due to El-Centro 1940 N-S, using :

a. Interpolation
b. Central Difference
c. Newmark Average Acceleration
d. Newmark Linear Acceleration