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Mata Kuliah : Dinamika Struktur & Pengantar Rekayasa Kegempaan
Kode : TSP – 302
SKS : 3 SKS

Single Degree of Freedom System

Arbitrary Force

Pertemuan - 4

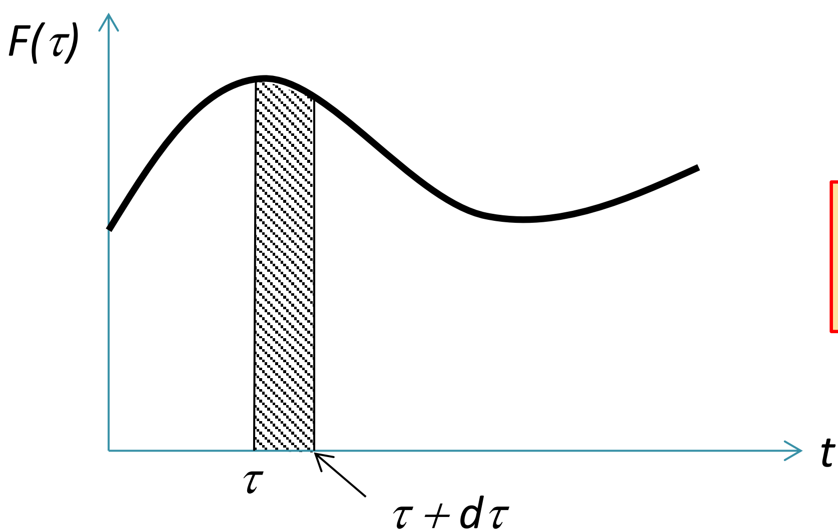
- TIU :
  - Mahasiswa dapat menjelaskan tentang teori dinamika struktur.
  - Mahasiswa dapat membuat model matematik dari masalah teknis yang ada serta mencari solusinya.

- TIK :
  - Mahasiswa mampu menghitung respon sistem struktur sederhana akibat beban luar sembarang

- Sub Pokok Bahasan :
  - Eksitasi Sembarang

Duhamel's Integral – Undamped System

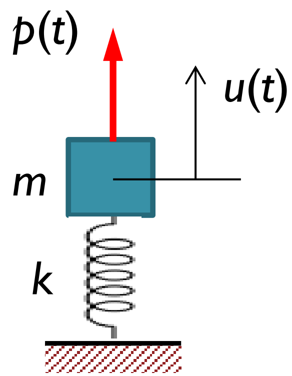
- An *impulsive loading* is a load which is applied during a short duration of time.
- The corresponding impulse of this type of load is defined as the product of the force and the time of its duration.

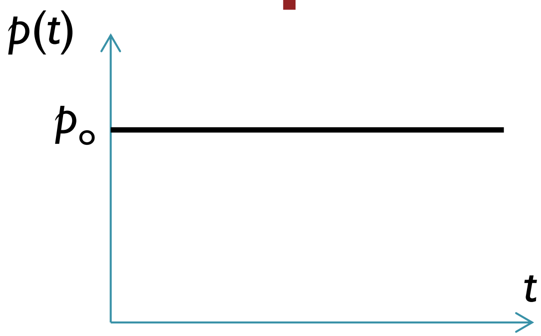


Using Duhamel's Integral, the displacement response at time t due to continuous $F(\tau)$ is :

$$u(t) = \frac{1}{m\omega_n} \int_0^t F(\tau) \sin \omega_n (t - \tau) d\tau \qquad (1)$$

Response to Step Forces





p(t) = p$_o$

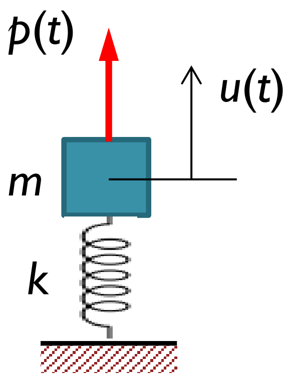
SDoF System Without Damping

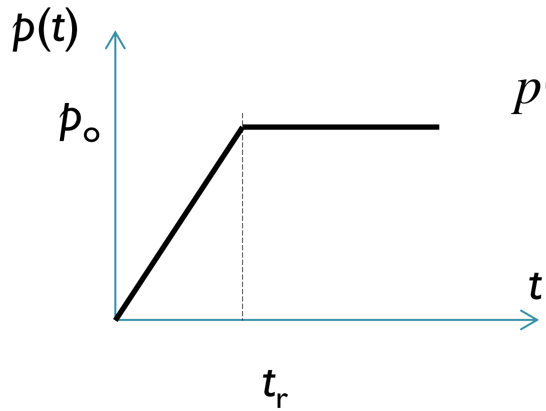
$$u(t) = (u_{st})_o(1 - \cos\omega_n t) = (u_{st})_o\left(1 - \cos\frac{2\pi t}{T_n}\right) \qquad \textbf{(2)} \qquad u_o = 2(u_{st})_o$$

SDoF System With Damping

$$u(t) = (u_{st})_o\left[1 - e^{-\xi\omega_n t}\left(\cos\omega_D t + \frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_D t\right)\right] \qquad \textbf{(3)}$$

Response to Step Force With Finite Rise Time





$$p(t)=\begin{cases} p_o(t/t_r) & t \le t_r \\ p_o & t \ge t_r \end{cases}$$

- If $t_r < T_n/4$: $u_o \approx 2(u_{st})_o$
- If $t_r > 3T_n$: $u_o \approx 2(u_{st})_o$
- If $t_r/T_n = 1,2,3, \ldots$ uo $= (u_{st})_o$

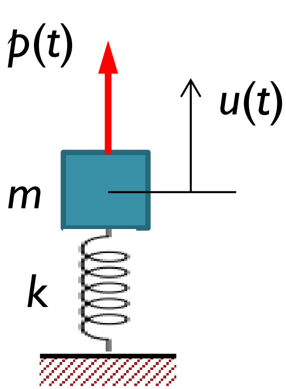
$$u(t)=(u_{st})_o\left(\frac{t}{t_r}-\frac{\sin\omega_n t}{\omega_n t_r}\right)$$

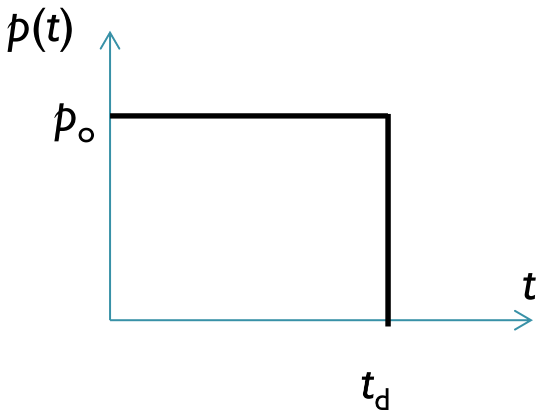
For $t \le t_r$ **(4)**

$$u(t)=(u_{st})_o\left\{1-\frac{1}{\omega_n t_r}\left[\sin\omega_n t-\sin\omega_n(t-t_r)\right]\right\}$$

For $t \ge t_r$ **(5)**

Response to Rectangular Pulse Force





$$p(t) = \begin{cases} p_o & t \le t_d \\ 0 & t \ge t_d \end{cases}$$

$$\frac{u(t)}{(u_{st})_o} = 1 - \cos \omega_n t = 1 - \cos \frac{2\pi t}{T_n}$$

For $t \le t_d$ **(6)**

$$\frac{u(t)}{(u_{st})_o} = \cos \omega_n (t - t_d) - \cos \omega_n t$$

For $t \ge t_d$ **(7)**

Response to Rectangular Pulse Force

- The maximum deformation is :

$$u_o = (u_{st})_o R_d = \frac{p_o}{k} R_d \qquad \textbf{(8)}$$

- With deformation response factor :

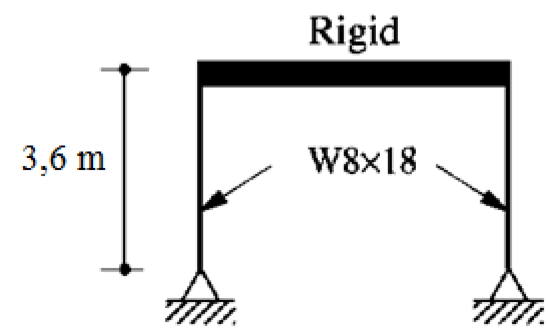
$$R_d = \begin{cases} 2\,sin\dfrac{\pi t_d}{T_n} & \text{for } t_d/T_n \le 1/2 \\ 2 & \text{for } t_d/T_n \ge 1/2 \end{cases} \qquad \textbf{(9)}$$

- The maximum value of the equivalent static force is :

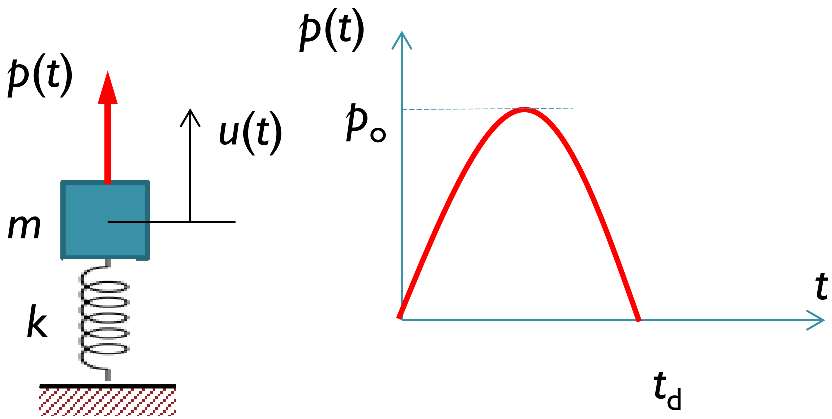
$$f_{So} = ku_o = p_o R_d \qquad \textbf{(10)}$$

Exercise

- A one-story, idealized as a 3,6 m high frame with two columns hinged at the base and a rigid beam, has a natural period of 0,5 sec. Each column is a wide-flange steel section W8x18. Its properties for bending about its major axis are I_x = 2,570 cm^4, S = I_x/c = 249 cm^3; E = 200 GPa. Neglecting damping, determine the maximum response of this frame due to a rectangular pulse force of amplitude 1,800 kgf and duration t_d = 0,2 sec. The response quantities of interest are displacement at the top of the frame and maximum bending stress in the column.



Response to Half Cycle Sine Pulse Force



$$p(t)=\begin{cases} p_o \sin(\pi t/t_d) & t \le t_d \\ 0 & t \ge t_d \end{cases}$$

For $t_d/T_n \neq \frac{1}{2}$

$$\frac{u(t)}{(u_{st})_o}=\frac{1}{1-(T_n/2t_d)^2}\left[\sin\left(\pi\frac{t}{t_d}\right)-\frac{T_n}{2t_d}\sin\left(2\pi\frac{t}{T_n}\right)\right] \quad \text{For } t \le t_d$$

$$\frac{u(t)}{(u_{st})_o}=\frac{(T_n/t_d)\cos(\pi t_d/T_n)}{(T_n/2t_d)^2-1}\sin\left[2\pi\left(\frac{t}{T_n}-\frac{1}{2}\frac{t_d}{T_n}\right)\right] \quad \text{For } t \ge t_d$$

- For $t_d/T_n = ½$

$$\frac{u(t)}{(u_{st})_o} = \frac{1}{2}\left(sin\frac{2\pi t}{T_n} - \frac{2\pi t}{T_n} cos\frac{2\pi t}{T_n} \right)$$

For $t \leq t_d$

$$\frac{u(t)}{(u_{st})_o} = \frac{\pi}{2} cos\, 2\pi\left(\frac{t}{T_n} - \frac{1}{2} \right)$$

For $t \geq t_d$

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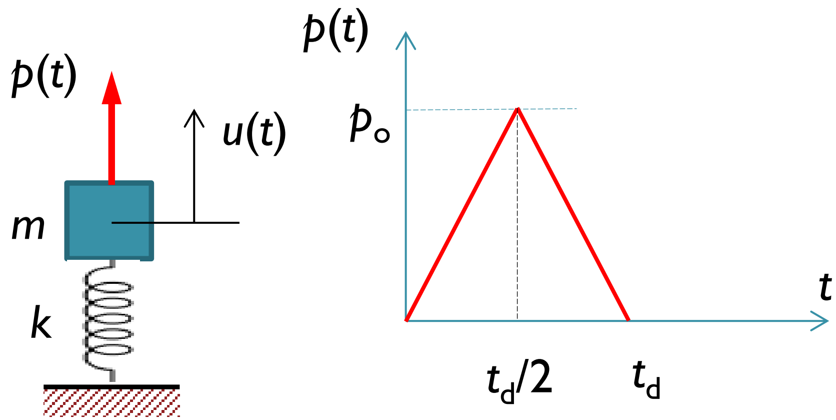
Response to Symmetrical Triangular Pulse Force



$$\frac{u(t)}{(u_{st})_o} = \begin{cases} 2\left(\frac{t}{t_d} - \frac{T_n}{2\pi t_d}\sin 2\pi\frac{t}{T_n}\right) & 0 \le t \le \frac{t_d}{2} \\ 2\left\{1 - \frac{t}{t_d} + \frac{T_n}{2\pi t_d}\left[2\sin\frac{2\pi}{T_n}\left(t - \frac{1}{2}t_d\right) - \sin 2\pi\frac{t}{T_n}\right]\right\} & \frac{t_d}{2} \le t \le t_d \\ 2\left\{\frac{T_n}{2\pi t_d}\left[2\sin\frac{2\pi}{T_n}\left(t - \frac{1}{2}t_d\right) - \sin\frac{2\pi}{T_n}(t - t_d) - \sin 2\pi\frac{t}{T_n}\right]\right\} & t \ge t_d \end{cases}$$