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Mata Kuliah : Dinamika Struktur & Pengantar Rekayasa Kegempaan
Kode : TSP – 302
SKS : 3 SKS

Single Degree of Freedom System

Forced Vibration

Pertemuan - 3

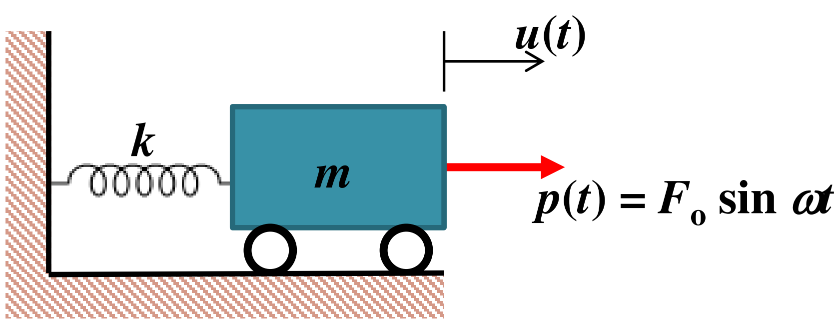
- TIU :
  - Mahasiswa dapat menjelaskan tentang teori dinamika struktur.
  - Mahasiswa dapat membuat model matematik dari masalah teknis yang ada serta mencari solusinya.

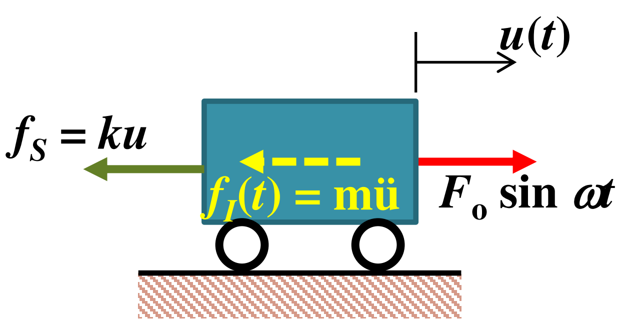
- TIK :
  - Mahasiswa mampu menghitung respon struktur dengan eksitasi harmonik dan eksitasi periodik

- Sub Pokok Bahasan :
  - Eksitasi Harmonik
  - Eksitasi Periodik

Undamped SDoF Harmonic Loading

- The impressed force $p(t)$ acting on the simple oscillator in the figure is assumed to be harmonic and equal to $F_o \sin \omega t$, where F_o is the amplitude or maximum value of the force and its frequency ω is called the *exciting frequency or forcing frequency.*





- The differential equation obtained by summing all the forces in the Free Body Diagram, is :

$$\boxed{m\ddot{u} + ku = F_o \sin \omega t} \qquad \textbf{(1)}$$

- The solution can be expresses as :

$$\boxed{u(t) = u_c(t) + u_p(t)} \qquad \textbf{(2)}$$

Complementary Solution

$u_c(t) = A \cos \omega_n t + B \sin \omega_n t$ **(3.a)**

Particular Solution

$u_p(t) = U \sin \omega t$ **(3.b)**

- Substituting Eq. (3.b) into Eq. (1) gives :

$$-m\omega^2 U + kU = F_o$$

- Or :

$$U = \frac{F_o}{k - m\omega^2} = \frac{F_o / k}{1 - \beta^2} \qquad \textbf{(4)}$$

- Which β represents the ratio of the applied forced frequency to the natural frequency of vibration of the system :

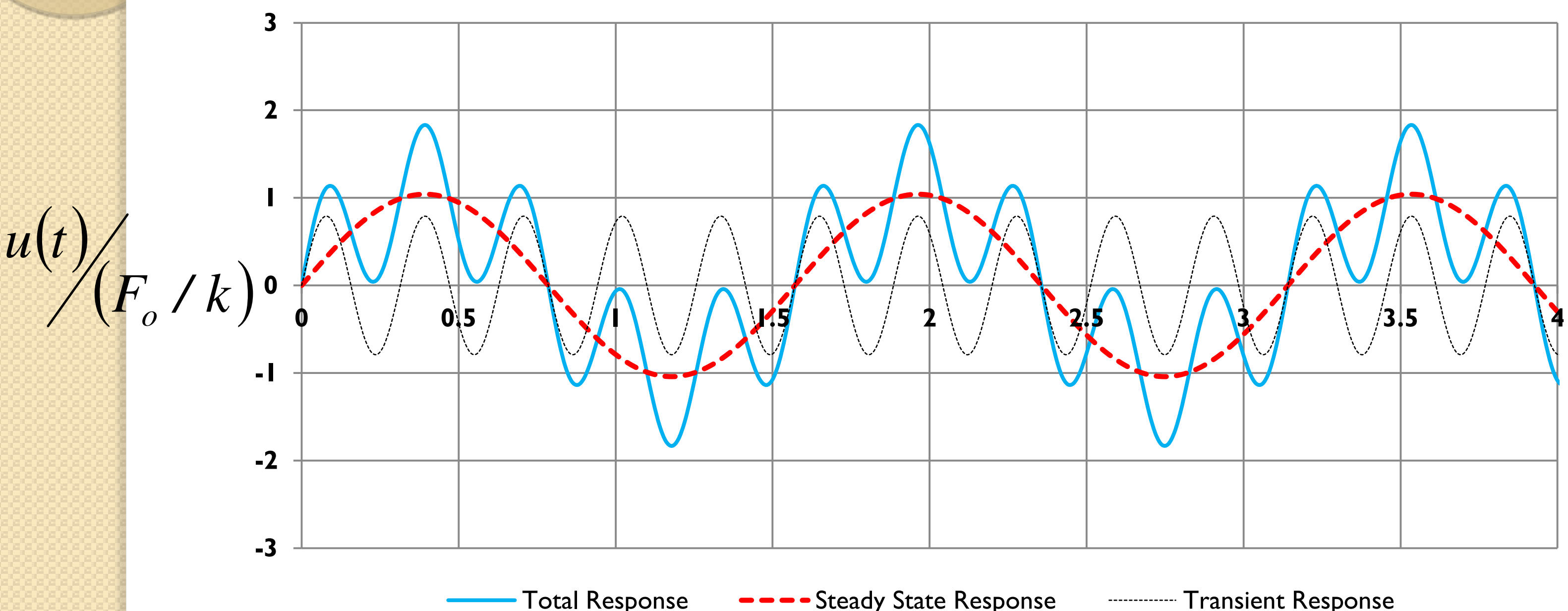
$$\boxed{\beta = \frac{\omega}{\omega_n}} \qquad \textbf{(5)}$$

- Combining Eq. (3.a & b) and (4) with Eq. (2) yields :

$$u(t) = A\cos\omega_n t + B\sin\omega_n t + \frac{F_o/k}{1-\beta^2}\sin\omega t \qquad \textbf{(6)}$$

- With initial conditions : $u = u(0)$ $\qquad \dot{u} = \dot{u}(0)$

$$\boxed{u(t) = \underbrace{u(0)\cos\omega_n t + \left[\frac{\dot{u}(0)}{\omega_n} - \frac{(F_o/k)\beta}{1-\beta^2}\right]\sin\omega_n t}_{\text{Transient Response}} + \underbrace{\frac{F_o/k}{1-\beta^2}\sin\omega t}_{\text{Steady State Response}}} \qquad \textbf{(7)}$$



$$u(0) = 0 \text{ and } \dot{u}(0) = \omega_n F_o / k$$

$$\omega / \omega_n = 0{,}20$$

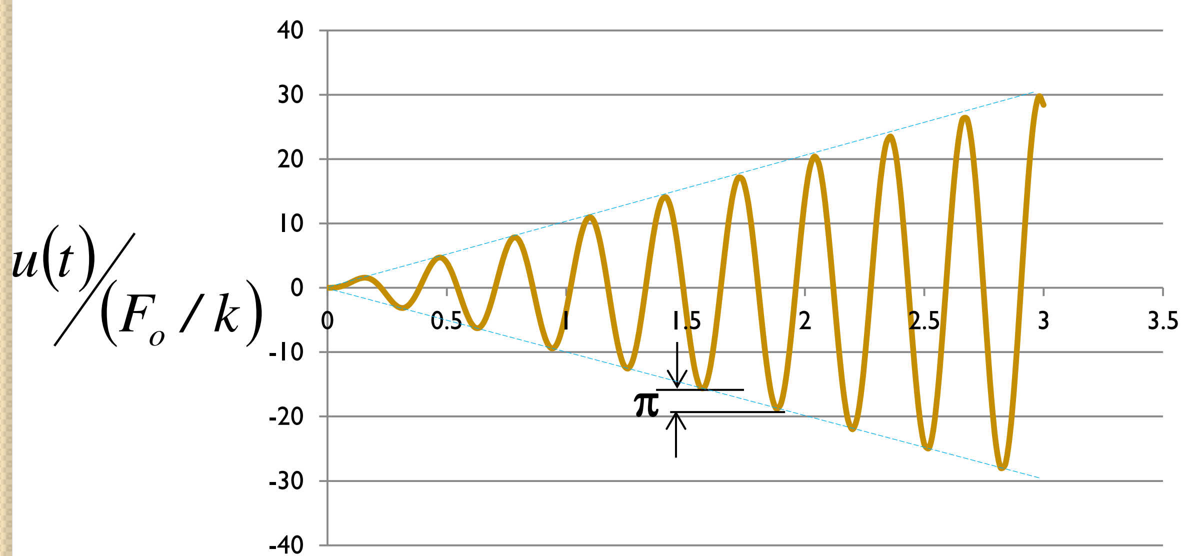
- Steady state response present because of the applied force, no matter what the initial conditions.
- Transient response depends on the initial displacement and velocity.
- Transient response exists even if $u(0) = \dot{u}(0) = 0$
- In which Eq. (7) specializes to

$$\boxed{u(t) = \frac{F_o/k}{1-\beta^2}\left(\sin \omega t - \beta \sin \omega_n t\right)} \qquad \textbf{(8)}$$

- It can be seen that when the forcing frequency is equal to natural frequency (β = 1), the amplitude of the motion becomes infinitely large.
- A system acted upon by an external excitation of frequency coinciding with the natural frequency is said to be at ***resonance***.

- If $\omega = \omega_n$ ($\beta = 1$), the solution of Eq. (1) becomes :

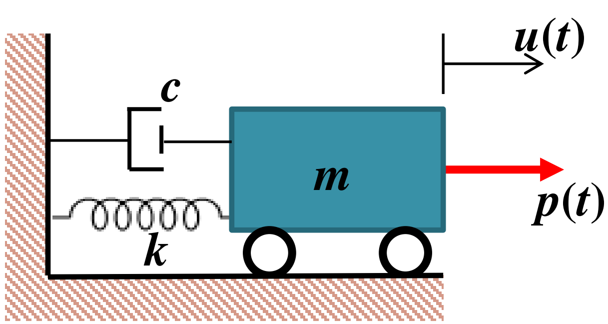
$$u(t) = -\frac{1}{2}\frac{F_o}{k}\left(\omega_n t \cos \omega_n t - \sin \omega_n t\right) \qquad \textbf{(9)}$$

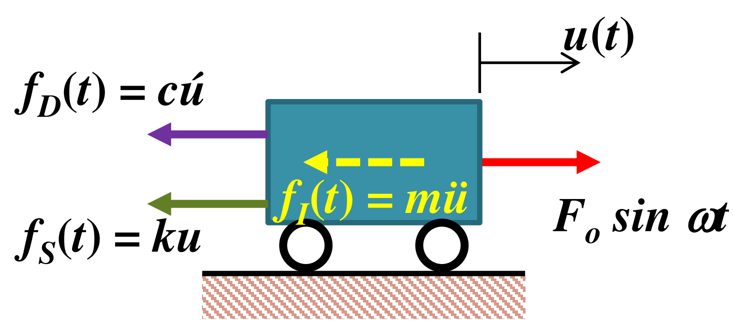


Damped SDoF Harmonic Loading

- Including viscous damping the differential equation governing the response of SDoF systems to harmonic loading is :

$$m\ddot{u} + c\dot{u} + ku = F_o \sin \omega t \tag{10}$$





- The complementary solution of Eq. (9) is :

$$u_c(t) = e^{-\xi\omega_n t}\left(A\cos\omega_D t + B\sin\omega_D t\right) \quad \textbf{(11)}$$

- The particular solution of Eq. (9) is :

$$u_p(t) = C\sin\omega t + D\cos\omega t \quad \textbf{(12)}$$

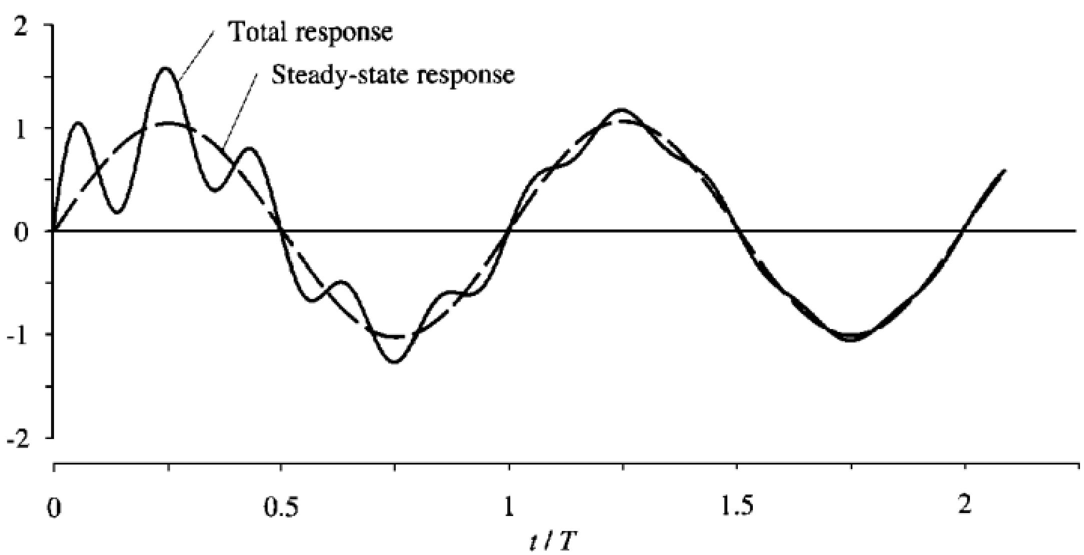
- Where :

$$C = \frac{F_o}{k}\frac{1-\beta^2}{\left[1-\beta^2\right]^2 + \left[2\xi\beta\right]^2} \quad \textbf{(13.a)}$$

$$D = \frac{F_o}{k}\frac{-2\xi\beta}{\left[1-\beta^2\right]^2 + \left[2\xi\beta\right]^2} \quad \textbf{(13.b)}$$

- The complete solution of Eq. (9) is :

$$u_c(t) = \underbrace{e^{-\xi\omega_n t}\left(A\cos\omega_D t + B\sin\omega_D t\right)}_{\text{Transient Response}} + \underbrace{C\sin\omega t + D\cos\omega t}_{\text{Steady State Response}} \qquad \textbf{(14)}$$

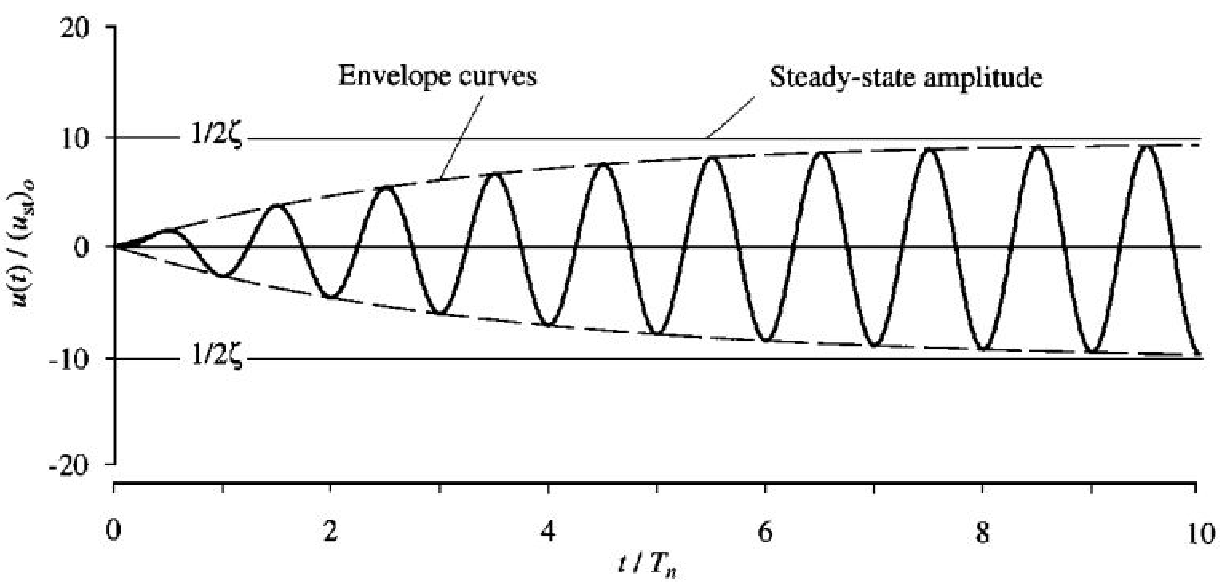


Response of damped system to harmonic force with
β = 0,2,
ξ = 0,05,
u(0) = 0,
ú(0) = $\omega_n F_o/k$

- The total response is shown by the solid line and the steady state response by the dashed line.
- The difference between the two is the transient response, which decays exponentially with time at a rate depending on β and ξ.
- After awhile, essentially the forced response remains, and called steady state response
- The largest deformation peak may occur before the system has reached steady state.

- If $\omega = \omega_n$ ($\beta = 1$), the solution of Eq. (10) becomes :

$$u(t) = \frac{F_o}{k}\frac{1}{2\xi}\left[e^{-\xi\omega_n t}\left(\cos\omega_D t + \frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_D t\right) - \cos\omega_n t\right] \quad \textbf{(15)}$$



- Considering only the steady state response, Eq. (12) & Eq. (13.a, b), can be rewritten as :

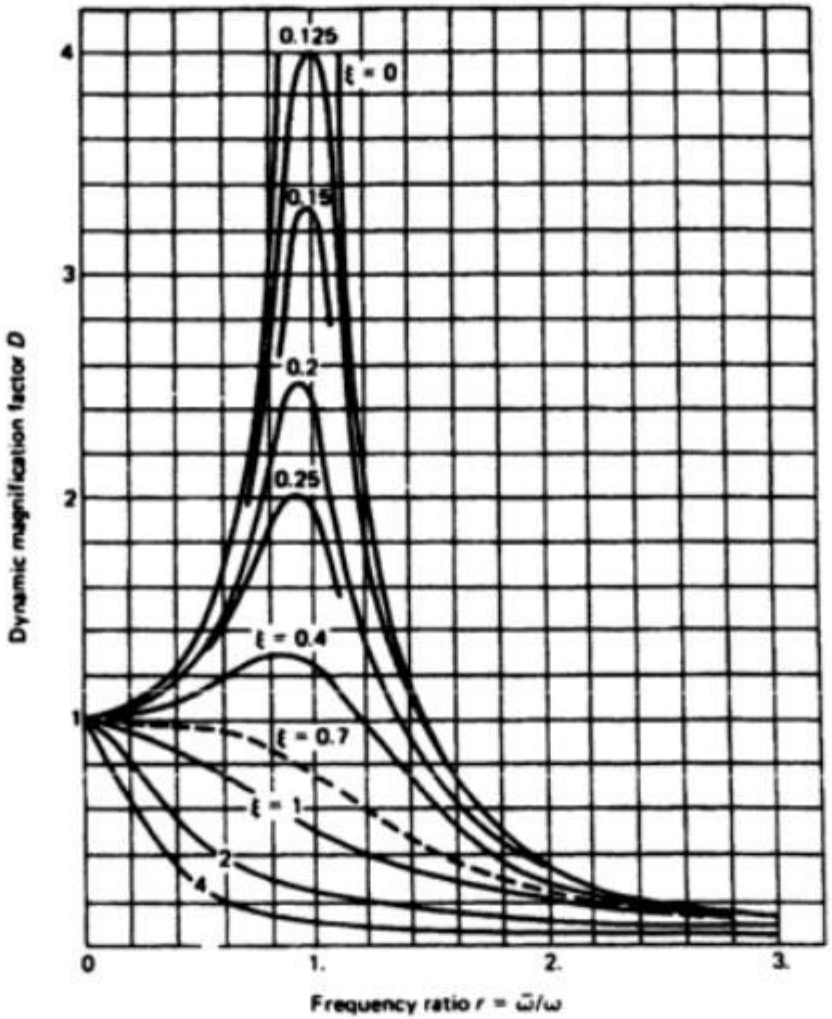
$$u(t) = U \sin(\omega t - \phi) \qquad \textbf{(16)}$$

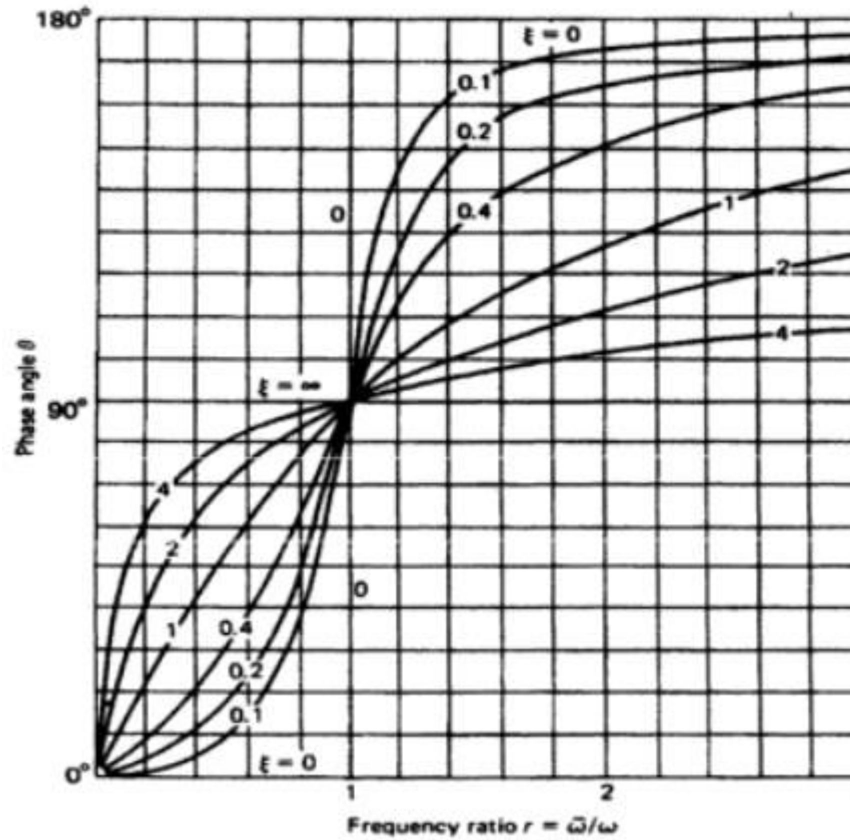
- Where :

$$U = \frac{F_o/k}{\sqrt{(1-\beta^2)^2 + (2\xi\beta)^2}} \qquad \boxed{\tan\phi = \frac{2\xi\beta}{1-\beta^2}} \qquad \textbf{(17)}$$

- Ratio of the steady state amplitude, *U* to the static deflection u_{st} ($=F_o/k$) is known as the dynamic magnification factor, *D* :

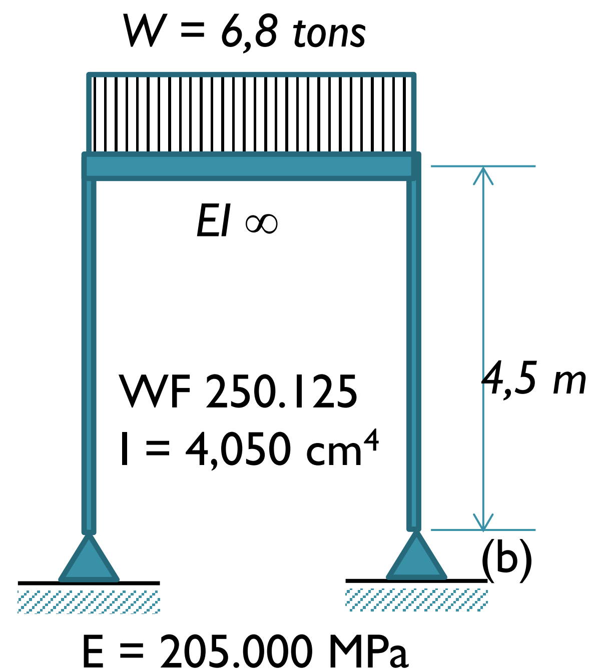
$$\boxed{D = \frac{U}{u_{st}} = \frac{1}{\sqrt{(1-\beta^2)^2 + (2\xi\beta)^2}}} \qquad \textbf{(18)}$$


0.125
ξ = 0
0.15
0.2
0.25
ξ = 0.4
ξ = 0.7
ξ = 1
2
4
Dynamic magnification factor D
Frequency ratio r = ω̄/ω


180°
90°
0°
ξ = 0
0.1
0.2
0.4
1
2
4
ξ = ∞
0
Phase angle θ
Frequency ratio r = ω̄/ω

Exercise

- The steel frame in the figure supports a rotating machine that exerts a horizontal force at the girder level p(t) = 100 sin 4 t kg. Assuming 5% of critical damping, determine : (a) the steady-state amplitude of vibration and (b) the maximum dynamic stress in the columns. Assume the girder is rigid



Response To Periodic Excitation

A periodic function can be separated into its harmonic components using Fourier Series

$$p(t) = a_0 + \sum_{j=1}^{\infty} a_j \cos(j\omega_0 t) + \sum_{j=1}^{\infty} b_j \sin(j\omega_0 t) \qquad \textbf{(19)}$$

Where :

$$a_0 = \frac{1}{T_0}\int_0^{T_0} p(t)\,dt$$

$$a_j = \frac{2}{T_0}\int_0^{T_0} p(t)\cos(j\omega_0 t)\,dt$$

$$b_j = \frac{2}{T_0}\int_0^{T_0} p(t)\sin(j\omega_0 t)\,dt$$

$$\omega_0 = \frac{2\pi}{T_0}$$

$$j = 1,2,3,\ldots$$

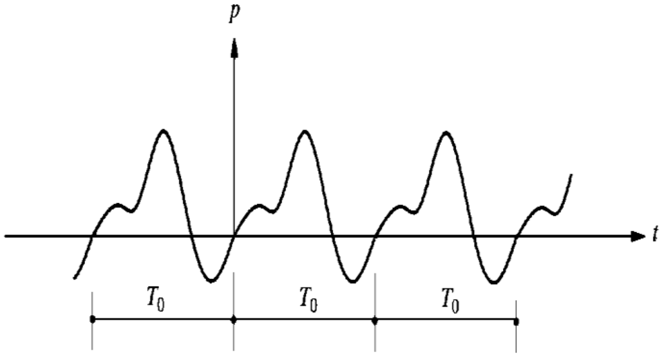


Figure 3.12.1 Periodic excitation.

- Response to periodic force is given by :

$$u(t) = \frac{a_0}{k} + \sum_{j=1}^{\infty} \frac{1}{k} \frac{1}{(1-\beta_j^2) + (2\zeta\beta_j)^2} \left\{ \left[a_j(2\zeta\beta_j) + b_j(1-\beta_j^2) \right] \sin(j\omega_0 t) + \left[a_j(1-\beta_j^2) - b_j(2\zeta\beta_j) \right] \cos(j\omega_0 t) \right\} \quad \textbf{(20)}$$

Example 3.8

The periodic force shown in Fig. E3.8a is defined by

$$p(t) = \begin{cases} p_o & 0 \le t \le T_0/2 \\ -p_o & T_0/2 \le t \le T_0 \end{cases}$$