DIRECT INTEGRATION

$$\frac{dy}{dx} = f(x)$$

$$\Downarrow$$

$$y = \int f(x)dx$$

SEPARATION OF VARIABLES

$$\frac{dy}{dx} = f(x)g(y)$$

$$\int \frac{dy}{g(y)} = \int f(x)dx$$

HOMOGENEOUS FUNCTION

$$\frac{dy}{dx} = \frac{M(x,y)}{N(x,y)}$$

$M(x, y)$ and $N(x, y)$

are homogeneous functions of the same degree

- Substitute $y = \nu x$
- Use separation of variable

$$\frac{dy}{dx} + P(x)y = Q(x)$$

Define Integrating Factor (IF) = $\exp\left[\int P(x)dx\right]$

$$\Longrightarrow \quad \frac{d}{dx}\left[\mathrm{IF} \cdot y\right] = \mathrm{IF} \cdot Q(x)$$

$$\Longrightarrow \quad y = \frac{1}{\mathrm{IF}} \int \mathrm{IF} \cdot Q(x)dx$$

$$\frac{dy}{dx} + p(x)y = q(x)y^n, n \neq 0, 1$$

Bernoullie differential equation looks similar to linear first order DE. It can be transformed into linear first order DE by performing several steps.

Divide the equation by y^n

$$y^{-n}\frac{dy}{dx} + p(x)y^{1-n} = q(x)$$

$$\nu = y^{-n}$$

Substitute: $\nu = y^{1-n}$

$$\frac{d\nu}{dx} + (1-n)p(x)\nu = (1-n)q(x)$$

- This is linear first order differential equation in ν
- Use the technique to solve linear first order DE
- Revert back to y once the differential equation for ν is solved

APPLICATION

- Applications of first order differential equation will be demonstrated
- Solutions will be obtained using one of the techniques learned in class
- Analysis will be performed with the aid of Geogebra
- We will consider 4 examples
  - Flexible rod
  - Tapered funnel
  - Compound interest
  - Population growth