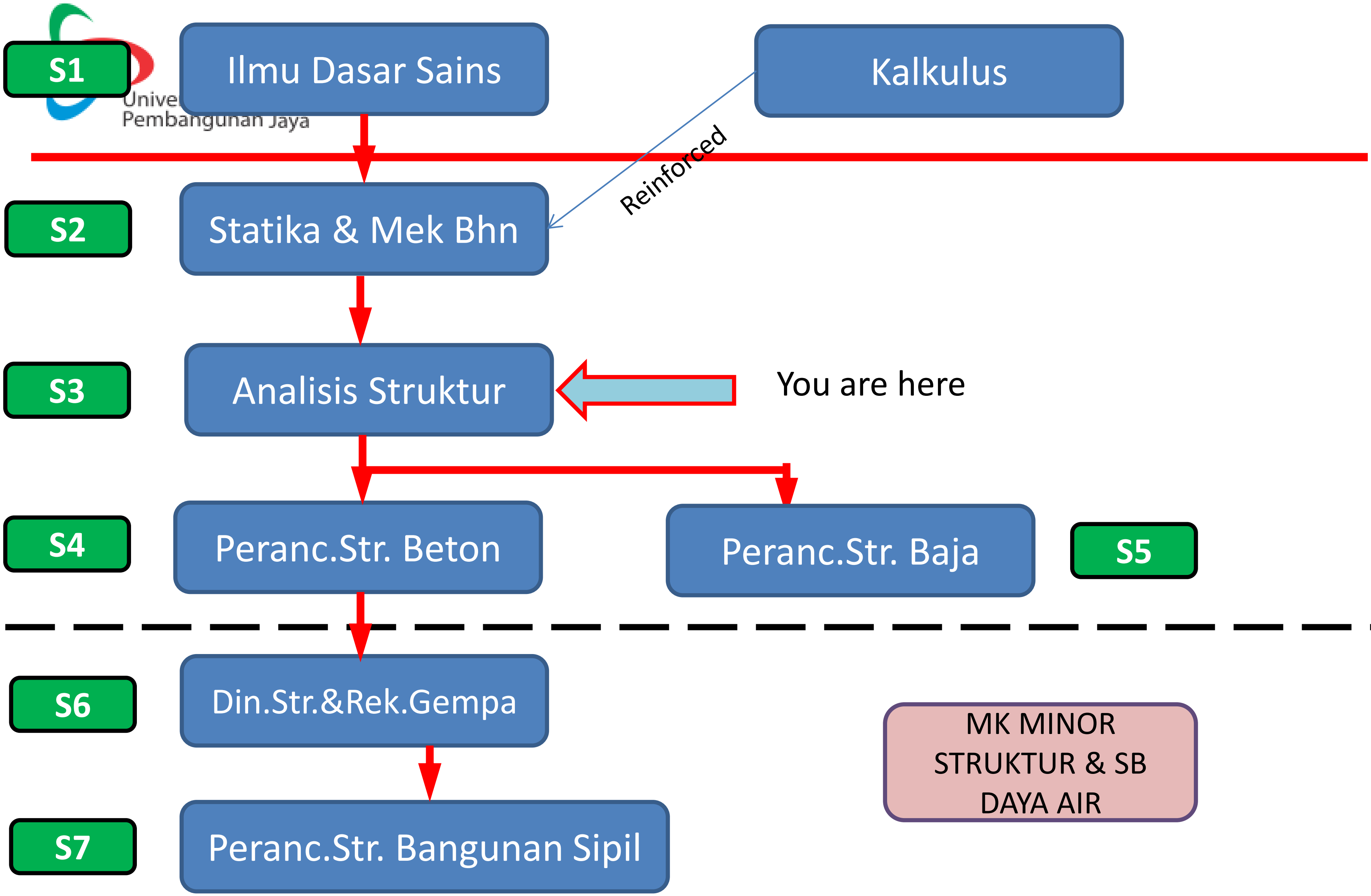
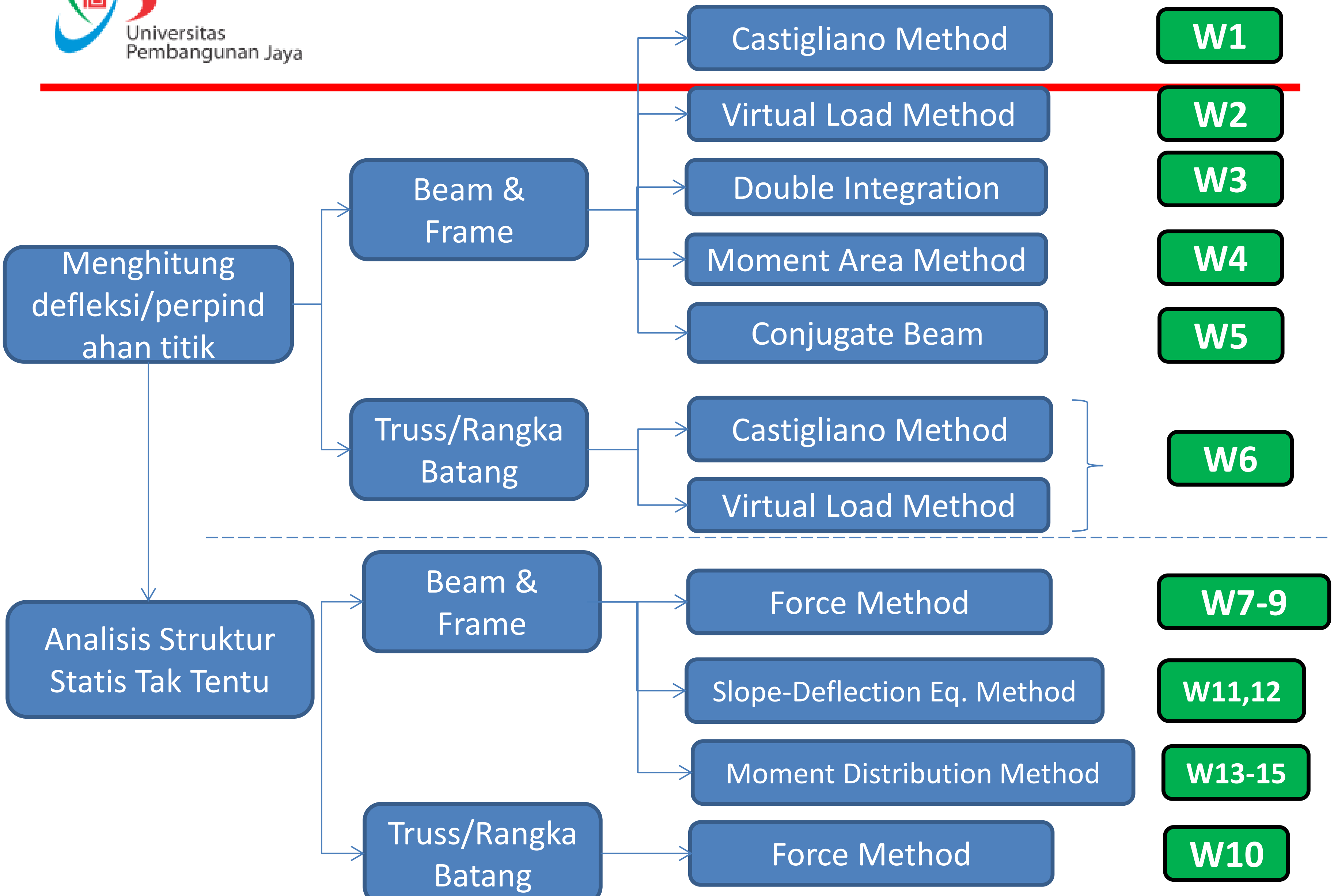

S1
Ilmu Dasar Sains
Kalkulus
Reinforced
S2
Statika & Mek Bhn
S3
Analisis Struktur
You are here
S4
Peranc.Str. Beton
Peranc.Str. Baja
S5
S6
Din.Str.&Rek.Gempa
MK MINOR
STRUKTUR & SB
DAYA AIR
S7
Peranc.Str. Bangunan Sipil
Pembangunan Jaya

Analisis Struktur (4SKS)

Universitas Pembangunan Jaya



Mata Kuliah : Analisis Struktur
Kode : CIV - 209
SKS : 4 SKS

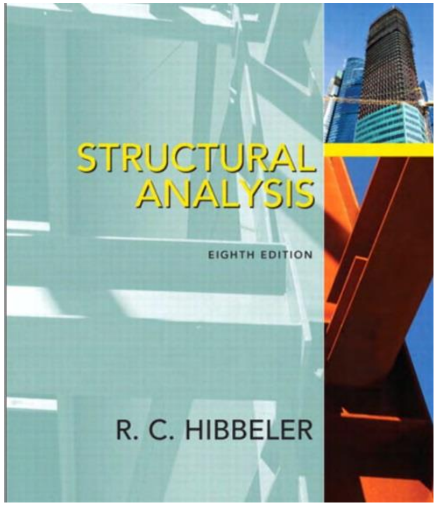
Prinsip Perpindahan Maya

Pertemuan – 1, 2

- Kemampuan Akhir yang Diharapkan
  - Mahasiswa dapat menjelaskan prinsip kerja dan Energi dalam perhitungan deformasi struktur

- Sub Pokok Bahasan :
  - Prinsip Dasar Metode Energi
  - Kerja dan Energi
  - Prinsip Konservasi Energi
  - Virtual work
  - Aplikasi kerja maya

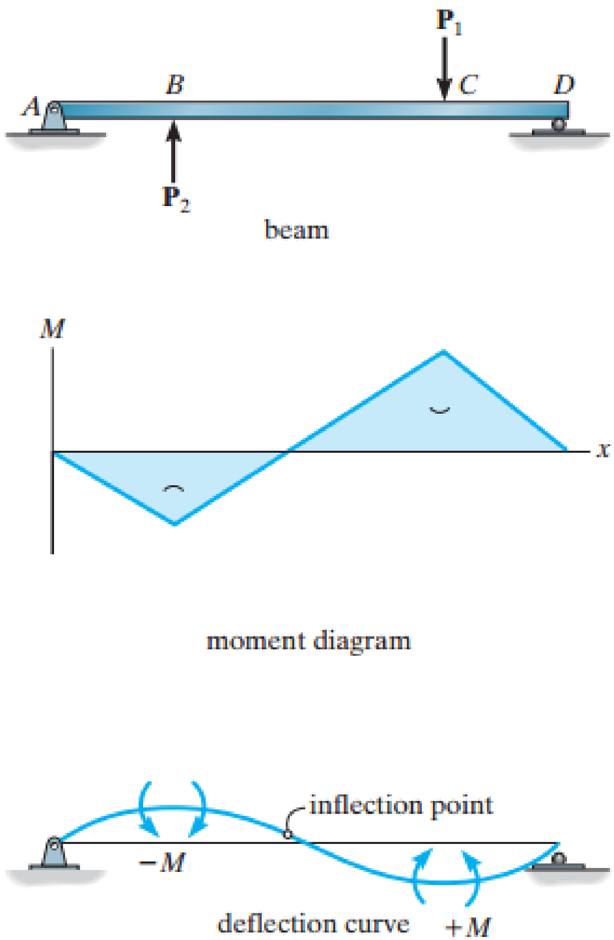
- Text Book :
  - **Hibbeler, R.C. (2010). Structural Analysis. 8th edition. Prentice Hall. ISBN : 978-0-13-257053-4**
  - West, H.H., (2002). Fundamentals of Structural Analysis. John Wiley & Sons, Inc. ISBN : 978-0471355564

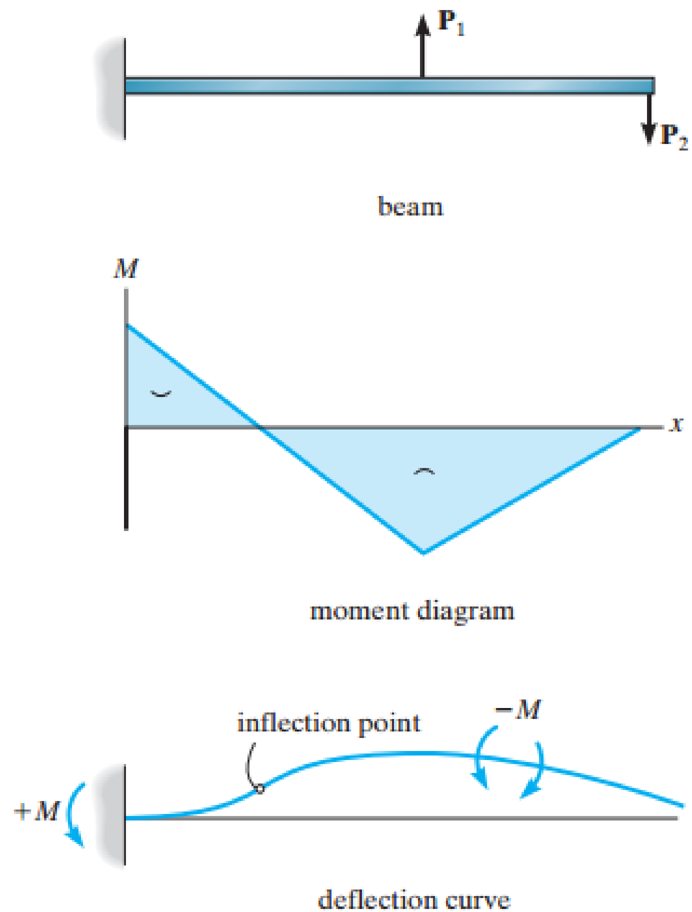


Deflections

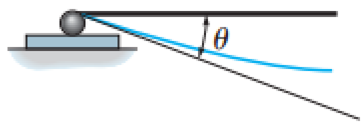
- Deflections of structures can occur from *various sources*, such as loads, temperature, fabrication errors, or settlement.
- In design, deflections must be limited in order to provide integrity and stability of roofs, and prevent cracking of attached brittle materials such as concrete, plaster or glass.

- Before the slope or displacement of a point on a beam or frame is determined, it is often helpful to sketch the deflected shape of the structure when it is loaded in order to partially check the results.
- This deflection diagram represents the ***elastic curve*** of points which defines the displaced position of the centroid of the cross section along the members.


P1
A
B
C
D
P2
beam
M
x
moment diagram
inflection point
−M
deflection curve
+M

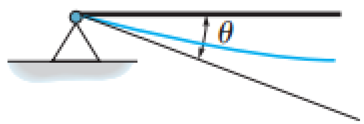

P1
P2
beam
M
x
moment diagram
inflection point
−M
+M
deflection curve

(1)



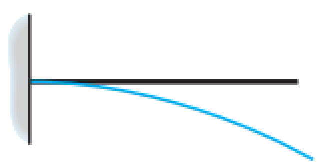
$\Delta = 0$
roller or rocker

(2)



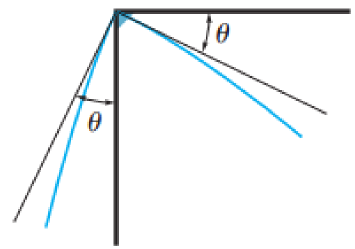
$\Delta = 0$
pin

(3)



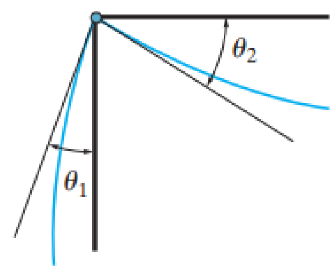
$\Delta = 0$
$\theta = 0$
fixed support

(4)



fixed-connected joint

(5)



pin-connected joint

- **Kerja**

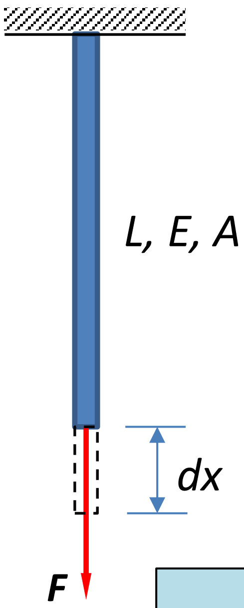
Prinsip Konservasi Energi (*conservation of energy principle) :*

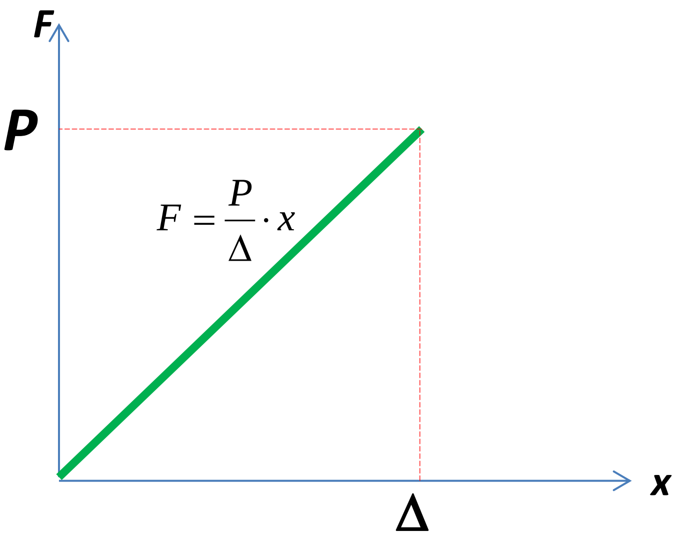
Kerja akibat seluruh gaya luar yang bekerja pada sebuah struktur (external forces) U_e, menyebabkan terjadinya gaya-gaya dalam pada struktur (internal work or strain energy) U_i seiring dengan deformasi yang terjadi pada struktur.

$$U_e = U_i \qquad (1)$$

Apabila tegangan yang terjadi tidak melebihi batas elastis material struktur tersebut, elastic strain energy akan mengembalikan bentuk struktur ke tahap awal sebelum terjadinya pembebanan, jika gaya-gaya luar yang bekerja dihilangkan.

External work-Axial force

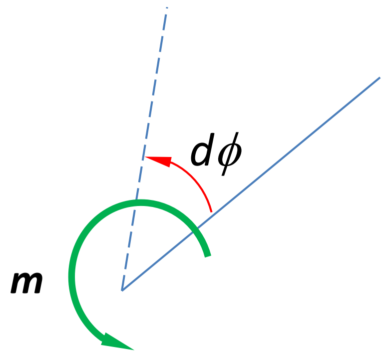


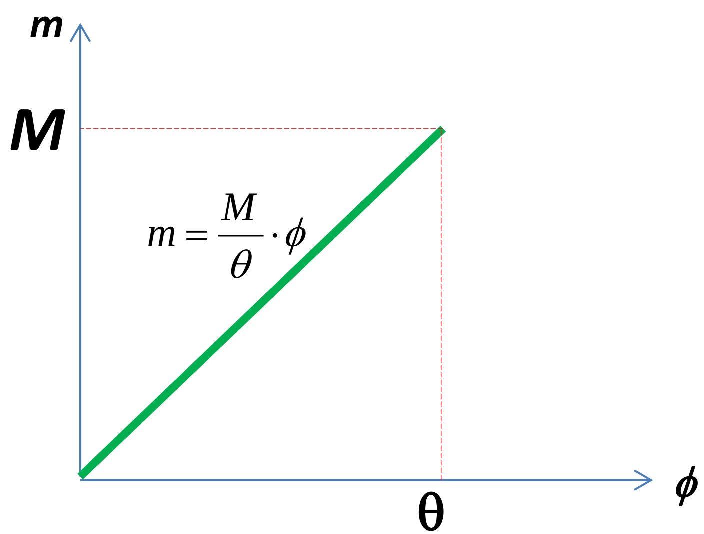


$$U_e = \int_0^{\Delta} F \cdot dx = \int_0^{\Delta} \frac{P}{\Delta} \cdot x \cdot dx = \frac{1}{2} P\Delta \qquad (2)$$

Kerja yang dilakukan oleh gaya luar P

External work-Bending moment

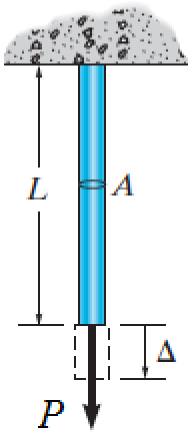




$$U_e = \int_0^\theta m \cdot d\phi = \int_0^\theta \frac{M}{\theta} \cdot \phi \cdot d\phi = \frac{1}{2} M\theta \qquad (3)$$

Kerja yang dilakukan oleh momen lentur M

Strain Energy – Axial force



Gaya P yang bekerja pada sebuah Bar seperti yang terlihat pada Gambar, dikonversikan menjadi *strain energy* yang menyebabkan pertambahan panjang pada batang sebesar Δ dan timbulnya tegangan σ. Mengingat Hukum Hooke : $\sigma = E\epsilon$.

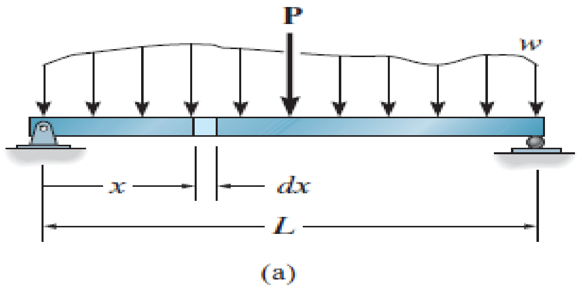
Maka persamaan defleksi dapat dituliskan menjadi:

$$\Delta = \frac{PL}{AE} \qquad \textbf{(4)}$$

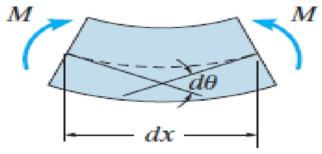
Subsitusikan persamaan 4 ke dalam persamaan 2, maka didapat energi regangan yang tersimpan dalam batang :

$$U_i = \frac{P^2 L}{2AE} \qquad \textbf{(5)}$$

Strain Energy – Bending Moment



(a)



(b)

$$d\theta = \frac{M}{EI}dx$$

Lihat Mekanika Bahan pt.11.

$$dU_i = \frac{1}{2}M \cdot d\theta = \frac{M^2}{2EI}dx$$

Dari pers.(3)

Energi regangan yang tersimpan pada balok :

$$U_i = \int_0^L \frac{M^2}{2EI}dx = \frac{M^2 L}{2EI} \qquad \textbf{(6)}$$

- Resume

	External Work, U_e	Internal Work, U_i
Axial Force	$\frac{1}{2}P\Delta$	$\frac{P^2L}{2AE}$
Bending Moment	$\frac{1}{2}M\theta$	$\frac{M^2L}{2EI}$

Castigliano Theorem

- Italian engineer Alberto Castigliano (1847 – 1884) developed a method of determining deflection of structures by strain energy method.
- His Theorem of the Derivatives of *Internal Work of Deformation* extended its application to the calculation of relative rotations and displacements between points in the structure and to the study of beams in flexure.

Castigliano's Theorem for Beam Deflection

- For linearly elastic structures, the partial derivative of the strain energy with respect to an applied force (or couple) is equal to the displacement (or rotation) of the force (or couple) along its line of action.

$$\Delta_i = \frac{\partial U_i}{\partial P_i} \quad \text{or} \quad \theta_i = \frac{\partial U_i}{\partial M_i} \tag{7}$$

- Subtitusi persamaan 6 ke persamaan 7 :

$$\Delta = \frac{\partial}{\partial P}\int_0^L \frac{M^2 dx}{2EI} = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} \qquad (8)$$

- Jika sudut rotasi θ, yang hendak dicari :

$$\theta = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} \qquad (9)$$

where

Δ = external displacement of the point caused by the real loads acting on the beam or frame.

P = external force applied to the beam or frame in the direction of Δ.

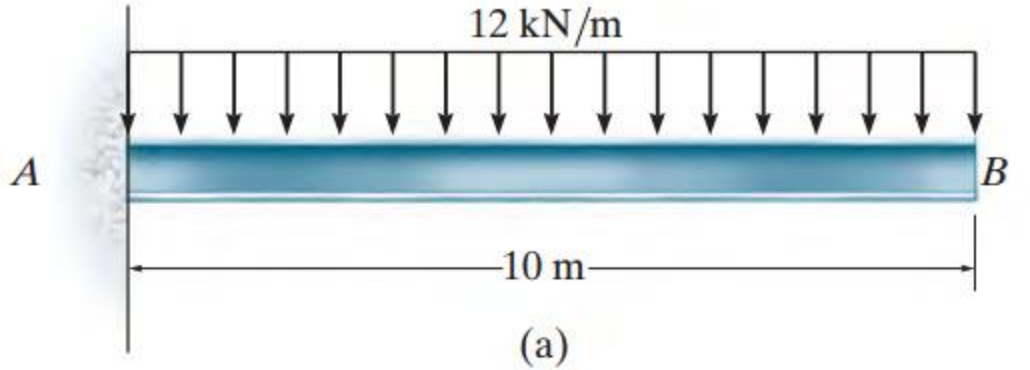
M = internal moment in the beam or frame, expressed as a function of x and caused by both the force P and the real loads on the beam.

E = modulus of elasticity of beam material.

I = moment of inertia of cross-sectional area computed about the neutral axis.

Example 1

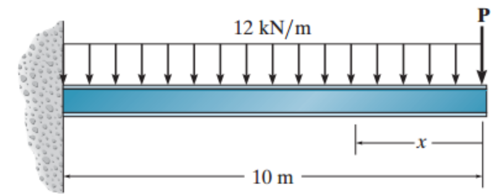
- Determine the displacement of point B of the beam shown in the Figure.
- Take E = 200 GPa, I = 500(10^6) mm^4.



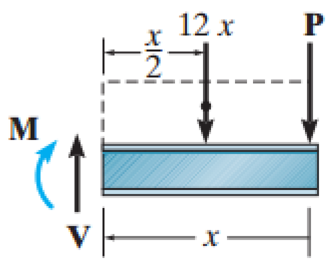
(a)

Example 1

- A vertical force P is placed on the beam at B



- Internal moments : (taken from the right side of the beam)



$$\Sigma M = 0$$

$$M + \left(12x \cdot \frac{x}{2}\right) + P \cdot x = 0$$

$$M = -6x^2 - Px$$

$$\boxed{\frac{\partial M}{\partial P} = -x}$$

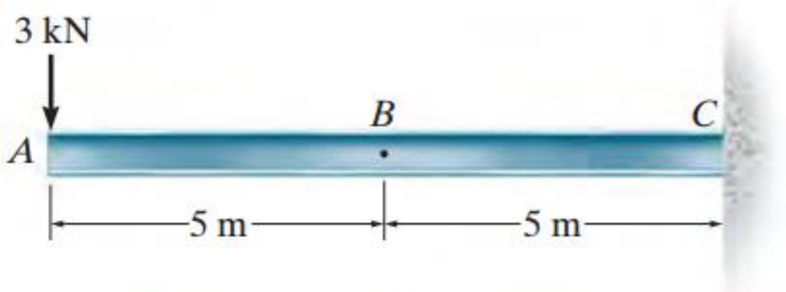
since $P = 0$ $\boxed{M = -6x^2}$

- From Castigliano's theorem :

$$\Delta_B = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^{10} \frac{\left(-6x^2\right)\left(-x\right)dx}{EI} = \frac{15 \cdot 10^3 kN \cdot m^3}{EI} = \underline{0{,}150m}$$

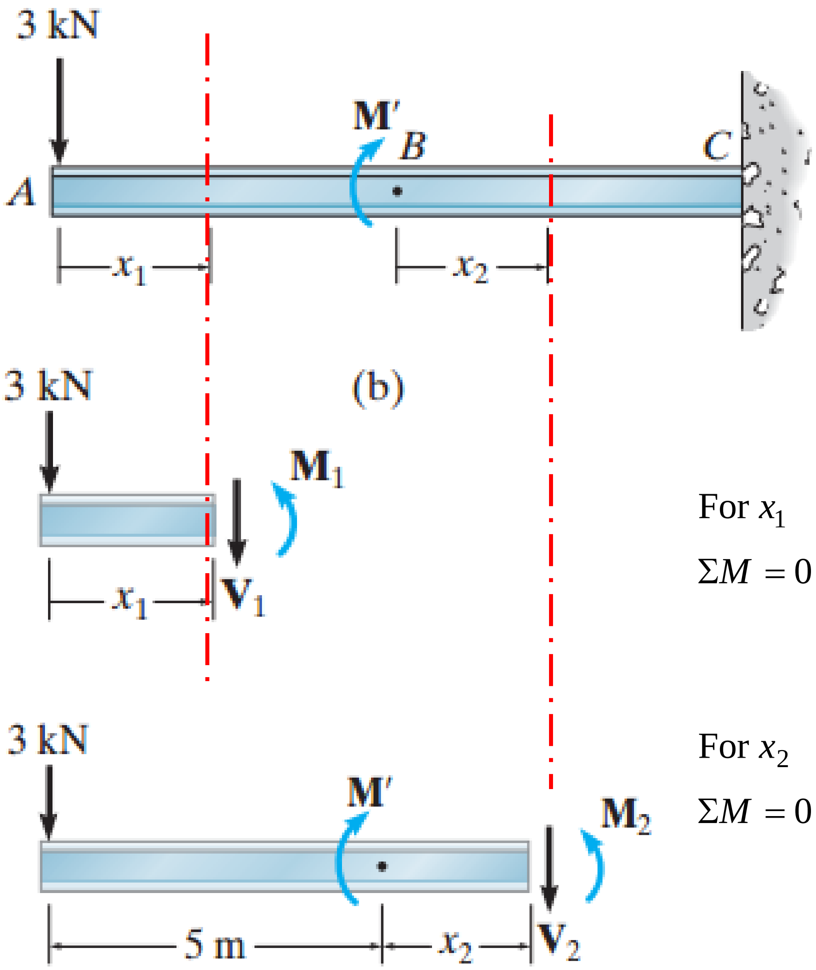
Example 2

- Determine the slope at point B of the beam shown in Figure.
- Take E = 200 GPa, I = $60(10^6)$ mm^4.



(a)

- Since the the slope at B is to be determined, an external couple M’ is placed on the beam at B.



For x_1

$$\Sigma M = 0 \qquad M_1 + 3x_1 = 0 \quad \Rightarrow \quad M_1 = -3x_1$$

$$\frac{\partial M_1}{\partial M'} = 0$$

For x_2

$$\Sigma M = 0 \qquad M_2 - M' + 3(5 + x_2) = 0 \quad \Rightarrow \quad M_2 = M' - 3(5 + x_2)$$

$$\frac{\partial M_2}{\partial M'} = 1$$

Setting *M'* = 0, its actual value, and using Castigliano Theorem, we have :

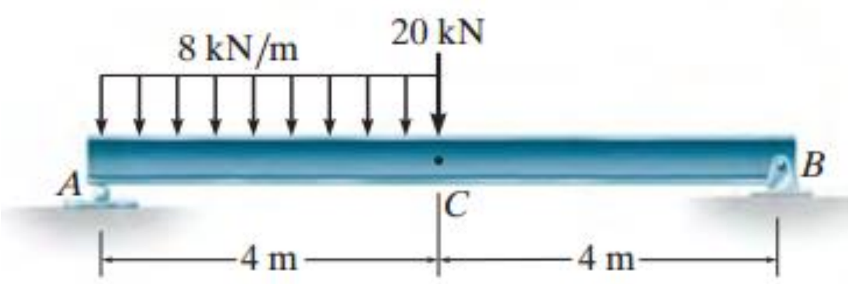
$$\theta_B = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \int_0^5 \frac{(-3x_1)(0)dx_1}{EI} + \int_0^5 \frac{-3(5+x_2)(1)dx_2}{EI} = -\frac{112{,}5\,\text{kN}\cdot\text{m}^2}{EI}$$

$$\theta_B = \frac{-112{,}5}{200\times 60} = \boxed{-9{,}375\cdot 10^{-3}\,\text{rad}}$$

The negative sign indicates that θ_B is opposite to the direction of the couple moment *M'*.

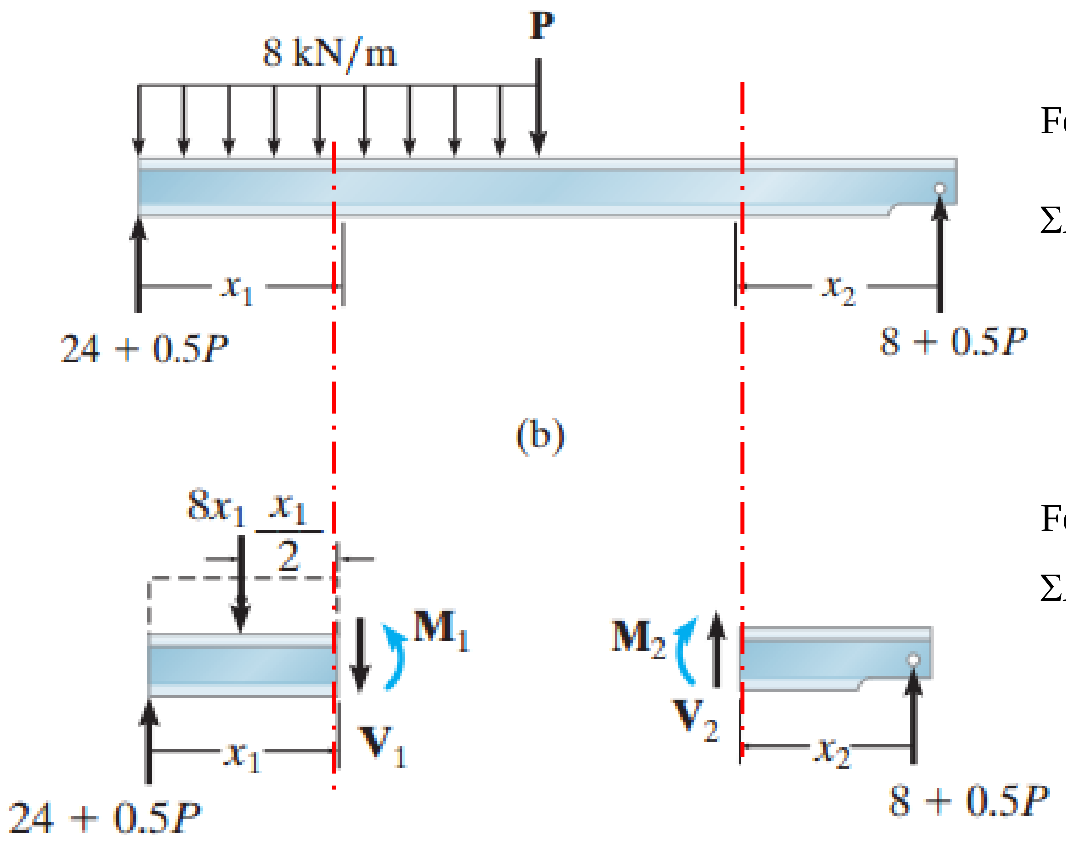
Example 3

- Determine the vertical displacement of point C of the beam.
- Take E = 200 GPa, I = $150(10^6)$ mm^4.



(a)

External Force P. A vertical force **P** is applied at point *C*. Later this force will be set equal to a fixed value of 20 kN.
Internal Moments M. In this case two *x* coordinates are needed for the integration, since the load is discontinuous at *C*.



For x_1

$$\Sigma M = 0 \qquad -(24+0{,}5P)x_1 + 8x_1\left(\frac{x_1}{2}\right) + M_1 = 0$$

$$\Rightarrow \quad M_1 = (24+0{,}5P)x_1 - 4x_1^{\,2}$$

$$\frac{\partial M_1}{\partial P} = 0{,}5x_1$$

For x_2

$$\Sigma M = 0 \qquad -M_2 + (8+0{,}5P)x_2 = 0$$

$$\Rightarrow \quad M_2 = (8+0{,}5P)x_2$$

$$\frac{\partial M_2}{\partial P} = 0{,}5x_2$$

- Applying Castigliano's Theorem. Setting P = 20 kN :

$$\Delta_{C_v} = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^4 \frac{\left(34x_1 - 4x_1{}^2\right)\left(0{,}5x_1\right)dx_1}{EI} + \int_0^4 \frac{\left(18x_2\right)\left(0{,}5x_2\right)dx_2}{EI}$$

$$= \frac{234{,}7\text{kN}\cdot\text{m}^3}{EI} + \frac{192\text{kN}\cdot\text{m}^3}{EI} = \frac{426{,}7\text{kN}\cdot\text{m}^3}{EI}$$

$$= \frac{426{,}7}{200\times150} = 0{,}0142\ \text{m} = 14{,}2\ \text{mm}$$

Soal Latihan (Chapter IX, Hibbeler)

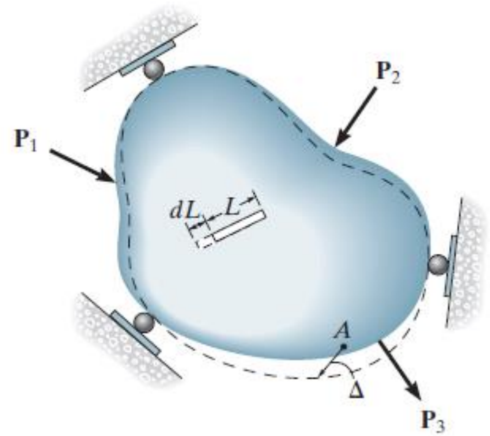
- 9.47
- 9.49
- 9.53
- 9.56
- 9.61
- 9.63
- 9.66
- 9.69
- 9.72
- 9.74
- 9.80
- 9.83
- 9.85
- 9.87
- 9.89
- 9.92
- 9.94
- 9.96
- 9.98

- Prinsip perpindahan maya (*virtual work*)

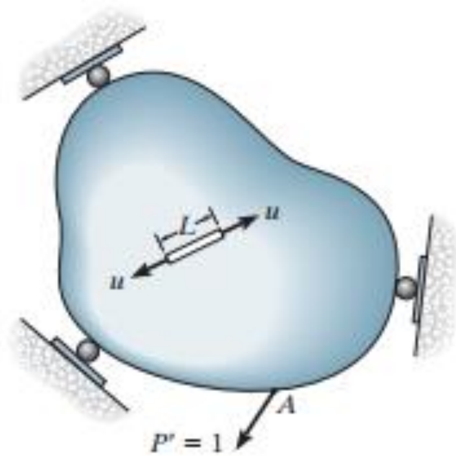
Prinsip ini dikembangkan oleh **John Bernoulli** pada tahun 1717 dan lebih dikenal dengan nama Unit Load Method.

General Statement :

- If we take a deformable structure of any shape or size and apply a series of external loads **P** to it, it will cause internal loads **u** at points throughout the structure.
- It is necessary that the external and internal loads be related by the *equations of equilibrium*.
- As a consequence of these loadings, external displacements Δ will occur at the **P** loads and internal displacements δ will occur at each point of internal load **u**.

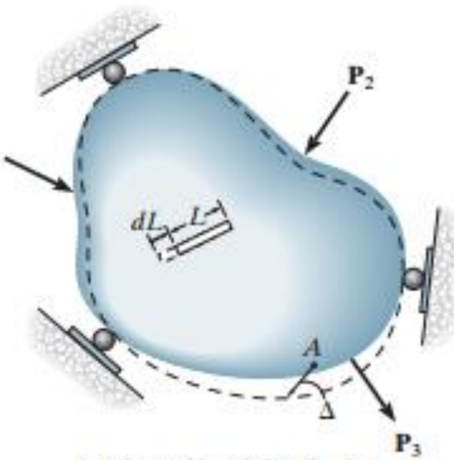


Gambar 2.1



Apply virtual load $P' = 1$

(a)



Apply real loads $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$

(b)

Gambar 2.2

$$\underset{\text{External Loads}}{\underset{\text{Work of}}{\Sigma P \Delta}} = \underset{\text{Internal Loads}}{\underset{\text{Work of}}{\Sigma u \delta}}$$

$$1.\Delta = \sum u\,.\,dL \qquad (1)$$

Virtual loading

Real displacement

Dimana :

P' = 1 = beban maya luar yang bekerja searah dengan Δ

Δ = perpindahan yang disebabkan oleh beban nyata

u = beban dalam maya yang bekerja dalam arah *dL*

dL = deformasi dalam benda yang disebabkan oleh beban nyata.

Dengan cara yang sama, apabila kita ingin menentukan besar sudut rotasi pada lokasi tertentu dari sebuah benda, kita dapat mengaplikasikan beban momen maya M' sebesar 1 satuan, lalu mengintegrasikannya dengan persamaan rotasi akibat beban momen nyata, sehingga :

$$1.\theta = \sum u_\theta \, . \, dL \qquad \textbf{(2)}$$

Virtual loading **Real displacement**

Dimana :

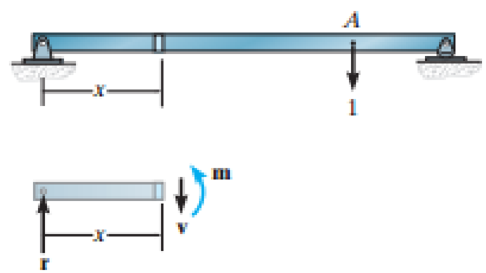
M' = 1 = beban maya luar yang bekerja searah dengan Δ

θ = perpindahan rotasi yang disebabkan oleh beban nyata

u_θ = kerja dalam maya yang bekerja dalam arah *dL*

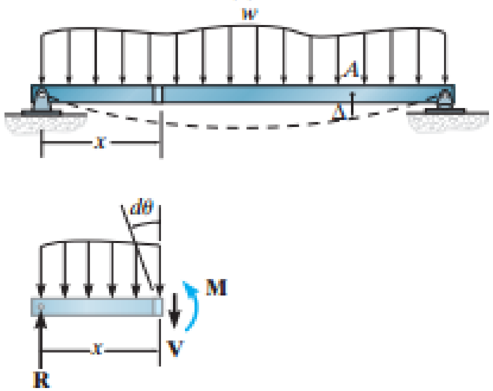
dL = deformasi dalam benda yang disebabkan oleh beban nyata.

- Kerja Maya Pada Balok/Frame



Apply virtual unit load to point A

(a)



Apply real load w

(b)

$$1 \cdot \Delta = \sum u\,dL$$

Virtual Loads

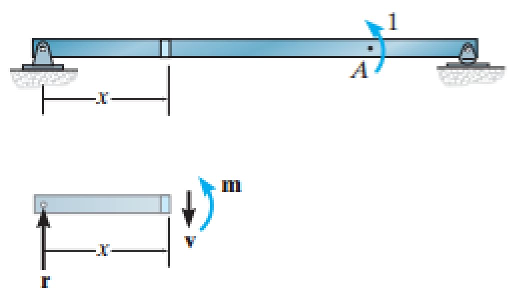
Real Displ.

$$u = m$$

$$dL = d\theta = \frac{M}{EI}dx$$

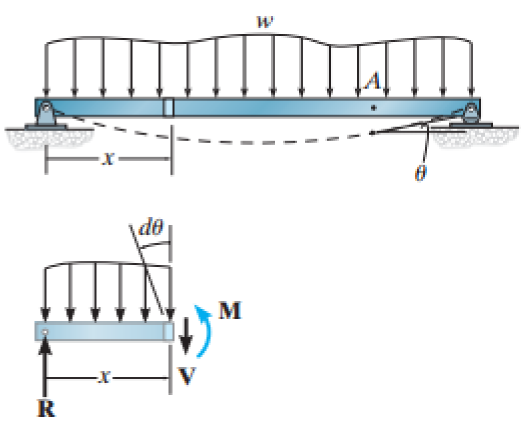
$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}dx \qquad \textbf{(3)}$$

- Kerja Maya Pada Balok/Frame



Apply virtual unit couple moment to point *A*

(a)



Apply real load *w*

$$1 \cdot \theta = \sum u_\theta dL$$

Virtual Loads

Real Displ.

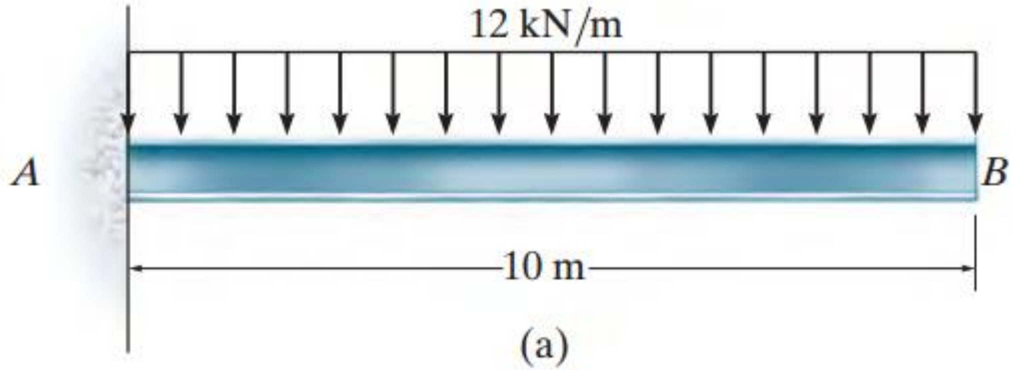
$$u = m_\theta$$

$$dL = d\theta = \frac{M}{EI} dx$$

$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI} dx \qquad \textbf{(4)}$$

Example 4

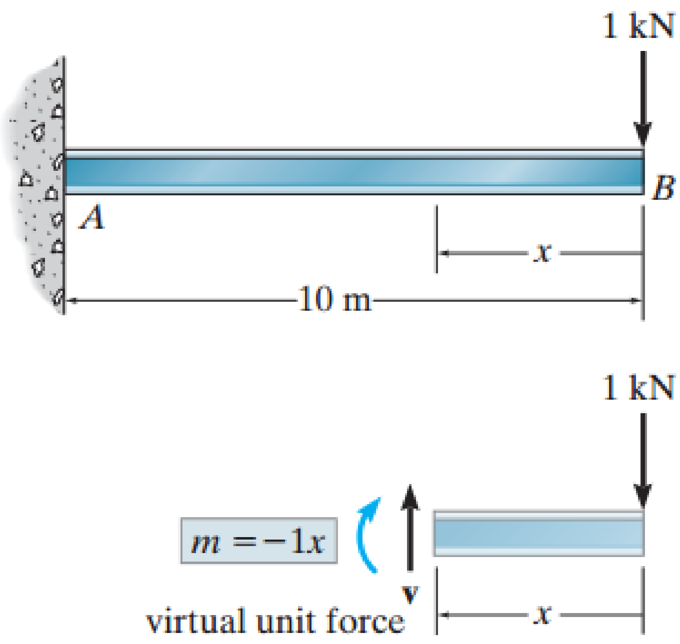
- Determine the displacement of point B of the beam shown in the Figure.
- Take E = 200 GPa, I = 500(10^6) mm^4.



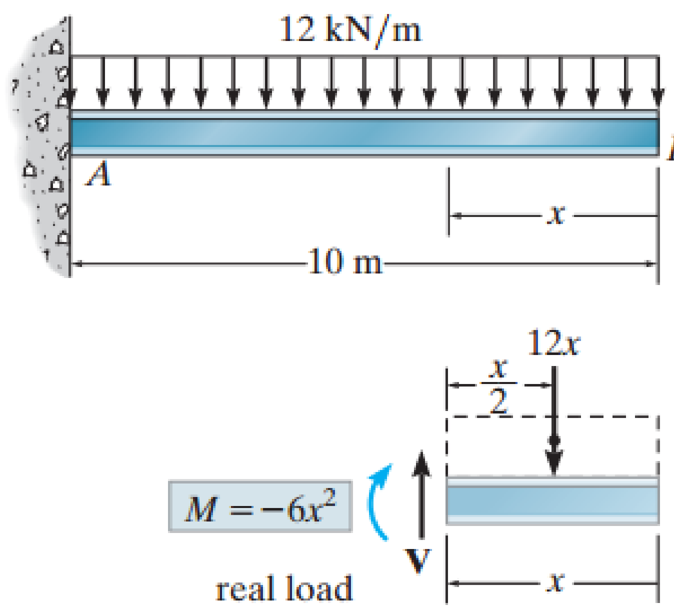
(a)

- The vertical displacement of point *B* is obtained by placing a virtual unit load of 1 kN at *B*.

Virtual Moment, m_θ



Real Moment, *M*

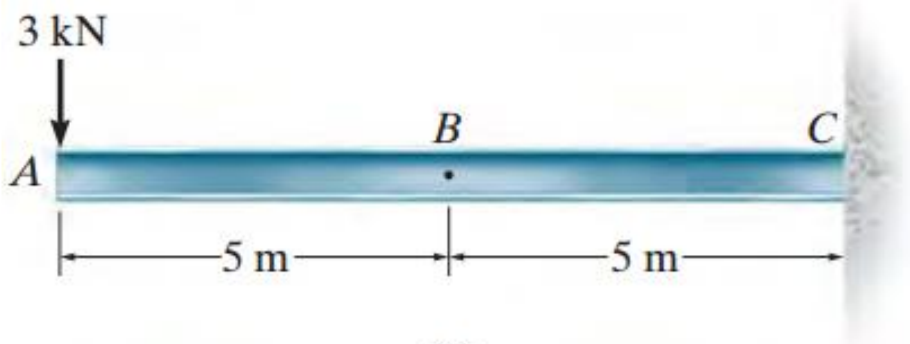


$$1\,\text{kN}\cdot\Delta_B = \int_0^L \frac{mM}{EI}\,dx = \int_0^{10} \frac{(-1x)(-6x^2)\,dx}{EI} = \frac{15.000\,\text{kN}^2\cdot\text{m}^3}{EI}$$

$$\Delta_B = \frac{15.000}{200\times 500} = \boxed{0{,}150\,\text{m} = 150\,\text{mm}}$$

Example 5

- Determine the slope at point B of the beam shown in Figure.
- Take E = 200 GPa, I = 60(10^6) mm^4.

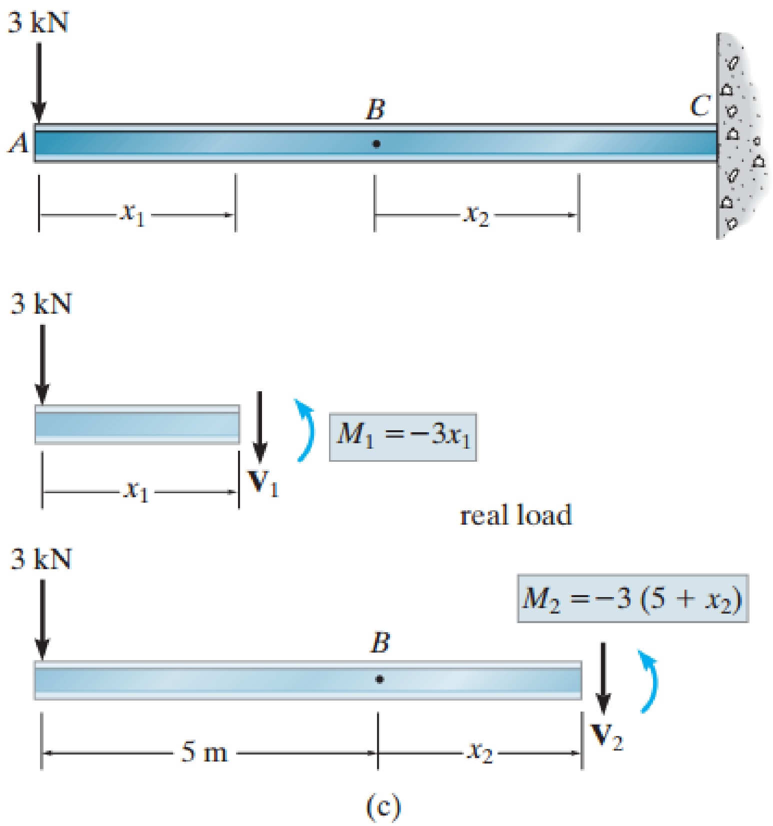


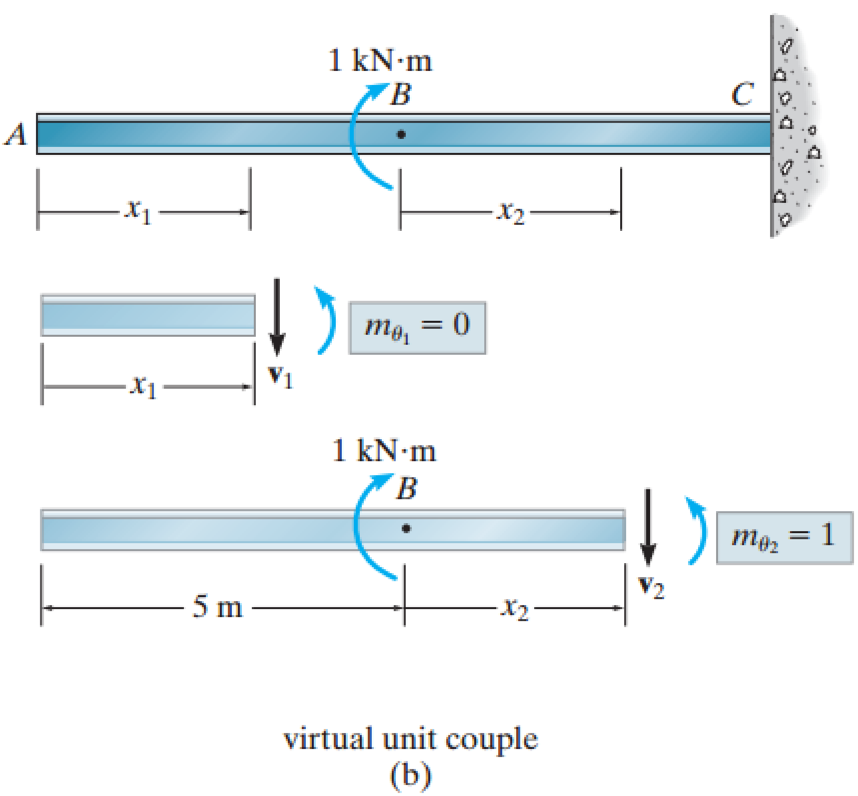
(a)

- The slope at B is determined by placing a virtual unit couple of 1 kN.m at B.
- Calculate virtual momen m_θ and real moment M

Real Moment, M

Virtual Moment, m_θ





- The slope at B, is thus given by :

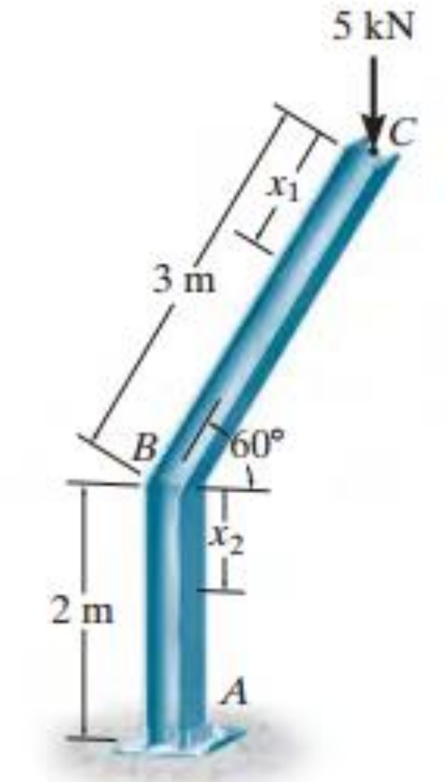
$$1\cdot\theta_B=\int_0^L \frac{m_\theta M}{EI}dx=\int_0^5 \frac{(0)(-3x_1)}{EI}dx_1+\int_0^5 \frac{(1)[-3(5+x_2)]}{EI}dx_2=\frac{-112{,}5}{EI}kN\cdot m^2$$

$$\theta_B=\frac{-112{,}5}{200\times 60}=-0{,}009375\,\text{rad}=\boxed{-9{,}375\cdot 10^{-3}\,\text{rad}}$$

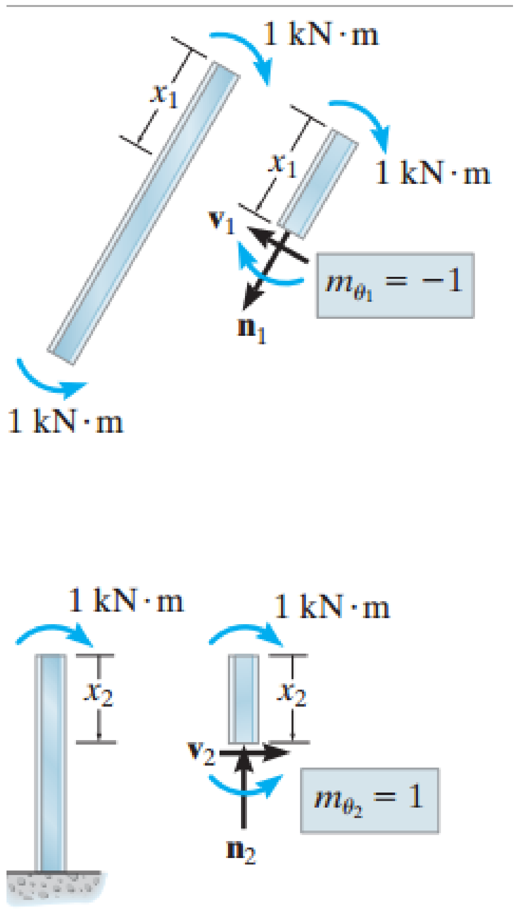
Example 7

- Determine the tangential rotation at point C of the frame shown in figure.
- Take E = 200 GPa,

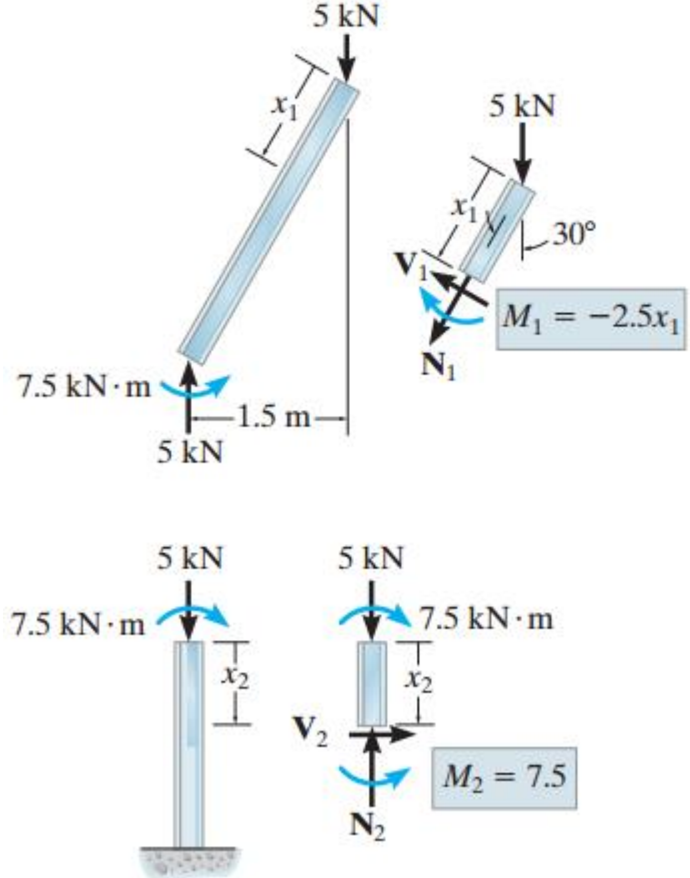
  $I = 15(10^6)\ mm^4$



(a)



virtual loads



real loads

$$1 \cdot \theta_C = \int_0^L \frac{m_\theta M}{EI} dx = \int_0^3 \frac{(-1)(-2{,}5x_1)dx_1}{EI} + \int_0^2 \frac{(1)(7{,}5)dx_2}{EI}$$

$$\theta_C = \frac{11{,}25}{EI} + \frac{15}{EI} = \frac{26{,}25\ \text{kN} \cdot \text{m}^2}{EI} = \frac{26{,}25}{200 \times 15} = 0{,}00875\ \text{rad}$$

Soal Latihan (Chapter IX, Hibbeler)

- **9.46**
- **9.48**
- **9.50**
- **9.51**
- **9.52**
- **9.54**
- **9.55**
- **9.57**
- **9.58**
- **9.59**
- **9.60**
- **9.62**
- **9.64**
- **9.65**
- **9.67**
- **9.68**
- **9.70**
- **9.71**
- **9.73**
- **9.75**
- **9.78**
- **9.79**
- **9.81**
- **9.82**
- **9.84**
- **9.86**
- **9.88**
- **9.90**
- **9.91**
- **9.93**
- **9.95**
- **9.97**