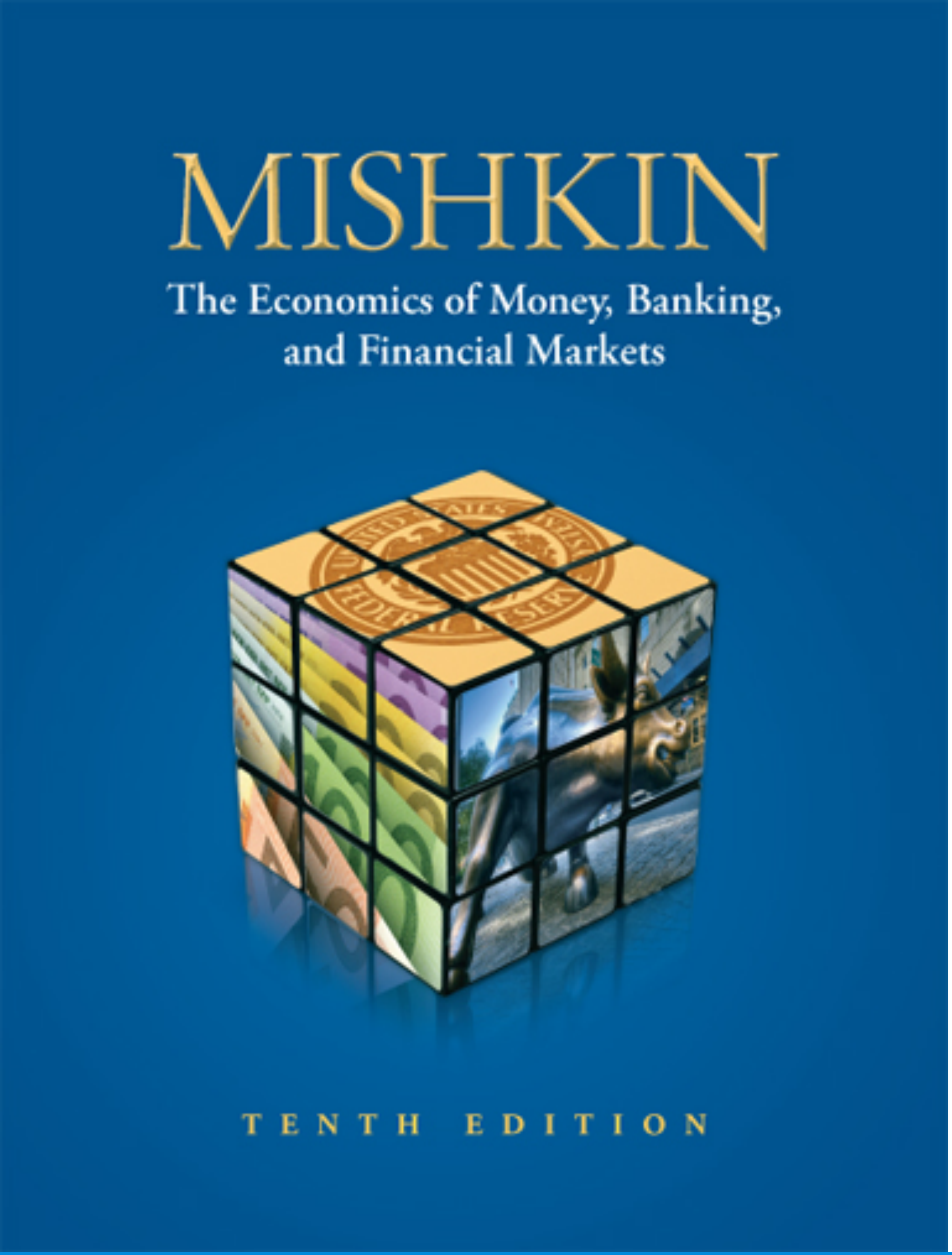

Chapter 4

Understanding Interest Rates

Measuring Interest Rates

- Present Value:
- A dollar paid to you one year from now is less valuable than a dollar paid to you today
- Why?
  - A dollar deposited today can earn interest and become \$1 x (1+i) one year from today.

Discounting the Future

Let $i = .10$

In one year $\$100 \text{ X } (1+ 0.10) = \$110$

In two years $\$110 \text{ X } (1 + 0.10) = \$121$

or $100 \text{ X } (1 + 0.10)^2$

In three years $\$121 \text{ X } (1 + 0.10) = \$133$

or $100 \text{ X } (1 + 0.10)^3$

In *n* years

$\$100 \text{ X } (1 + i)^n$

Simple Present Value

$$PV = \text{today's (present) value}$$

$$CF = \text{future cash flow (payment)}$$

$$i = \text{the interest rate}$$

$$PV = \frac{CF}{(1+i)^n}$$

Time Line

•Cannot directly compare payments scheduled in different points in the time line

	\$100	\$100	\$100	\$100
Year	0	1	2	n
PV	100	100/(1+i)	$100/(1+i)^2$	$100/(1+i)^n$

Four Types of Credit Market Instruments

- Simple Loan
- Fixed Payment Loan
- Coupon Bond
- Discount Bond

Yield to Maturity

- The interest rate that equates the present value of cash flow payments received from a debt instrument with its value today

Simple Loan

$$PV = \text{amount borrowed} = \$100$$

$$CF = \text{cash flow in one year} = \$110$$

$$n = \text{number of years} = 1$$

$$\$100 = \frac{\$110}{(1+i)^1}$$

$$(1+i)\ \$100 = \$110$$

$$(1+i) = \frac{\$110}{\$100}$$

$$i = 0.10 = 10\%$$

For simple loans, the simple interest rate equals the yield to maturity

Fixed Payment Loan

The same cash flow payment every period throughout the life of the loan

LV = loan value

FP = fixed yearly payment

n = number of years until maturity

$$LV = \frac{FP}{1+i} + \frac{FP}{(1+i)^2} + \frac{FP}{(1+i)^3} + \ldots + \frac{FP}{(1+i)^n}$$

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Coupon Bond

Using the same strategy used for the fixed-payment loan:

P = price of coupon bond

C = yearly coupon payment

F = face value of the bond

n = years to maturity date

$$P = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \ldots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n}$$

Table 1 Yields to Maturity on a 10%-Coupon-Rate Bond Maturing in Ten Years (Face Value = $1,000)

Yields to Maturity on a 10%-Coupon-Rate Bond Maturing in Ten Years (Face Value = $1,000)

Price of Bond ($)	Yield to Maturity (%)
1,200	7.13
1,100	8.48
1,000	10.00
900	11.75
800	13.81

- When the coupon bond is priced at its face value, the yield to maturity equals the coupon rate
- The price of a coupon bond and the yield to maturity are negatively related
- The yield to maturity is greater than the coupon rate when the bond price is below its face value

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Consol or Perpetuity

- A bond with no maturity date that does not repay principal but pays fixed coupon payments forever

$$P = C / i_c$$

$$P_c = \text{price of the consol}$$

$$C = \text{yearly interest payment}$$

$$i_c = \text{yield to maturity of the consol}$$

$$\text{can rewrite above equation as this}: i_c = C / P_c$$

For coupon bonds, this equation gives the current yield, an easy to calculate approximation to the yield to maturity

Discount Bond

For any one year discount bond

$$i = \frac{F - P}{P}$$

F = Face value of the discount bond

P = current price of the discount bond

The yield to maturity equals the increase in price over the year divided by the initial price.

As with a coupon bond, the yield to maturity is negatively related to the current bond price.

The Distinction Between Interest Rates and Returns

- Rate of Return: The payments to the owner plus the change in value expressed as a fraction of the purchase price

$$\text{RET} = \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$

$$\text{RET} = \text{return from holding the bond from time } t \text{ to time } t+1$$

$$P_t = \text{price of bond at time } t$$

$$P_{t+1} = \text{price of the bond at time } t+1$$

$$C = \text{coupon payment}$$

$$\frac{C}{P_t} = \text{current yield} = i_c$$

$$\frac{P_{t+1} - P_t}{P_t} = \text{rate of capital gain} = g$$

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The Distinction Between Interest Rates and Returns (cont'd)

- The return equals the yield to maturity only if the holding period equals the time to maturity
- A rise in interest rates is associated with a fall in bond prices, resulting in a capital loss if time to maturity is longer than the holding period
- The more distant a bond's maturity, the greater the size of the percentage price change associated with an interest-rate change

The Distinction Between Interest Rates and Returns (cont'd)

- The more distant a bond's maturity, the lower the rate of return the occurs as a result of an increase in the interest rate
- Even if a bond has a substantial initial interest rate, its return can be negative if interest rates rise

Table 2 One-Year Returns on Different-Maturity 10%-Coupon-Rate Bonds When Interest Rates Rise from 10% to 20%

One-Year Returns on Different-Maturity 10%-Coupon-Rate Bonds When Interest Rates Rise from 10% to 20%

(1) Years to Maturity When Bond Is Purchased	(2) Initial Current Yield (%)	(3) Initial Price ($)	(4) Price Next Year* ($)	(5) Rate of Capital Gain (%)	(6) Rate of Return (2 + 5) (%)
30	10	1,000	503	−49.7	−39.7
20	10	1,000	516	−48.4	−38.4
10	10	1,000	597	−40.3	−30.3
5	10	1,000	741	−25.9	−15.9
2	10	1,000	917	−8.3	+1.7
1	10	1,000	1,000	0.0	+10.0

*Calculated with a financial calculator using Equation 3.

Interest-Rate Risk

- Prices and returns for long-term bonds are more volatile than those for shorter-term bonds
- There is no interest-rate risk for any bond whose time to maturity matches the holding period

The Distinction Between Real and Nominal Interest Rates

- **Nominal interest rate** makes no allowance for inflation
- **Real interest rate** is adjusted for changes in price level so it more accurately reflects the cost of borrowing
- Ex ante real interest rate is adjusted for expected changes in the price level
- Ex post real interest rate is adjusted for actual changes in the price level

Fisher Equation

$$i = i_r + \pi^e$$

i = nominal interest rate

i_r = real interest rate

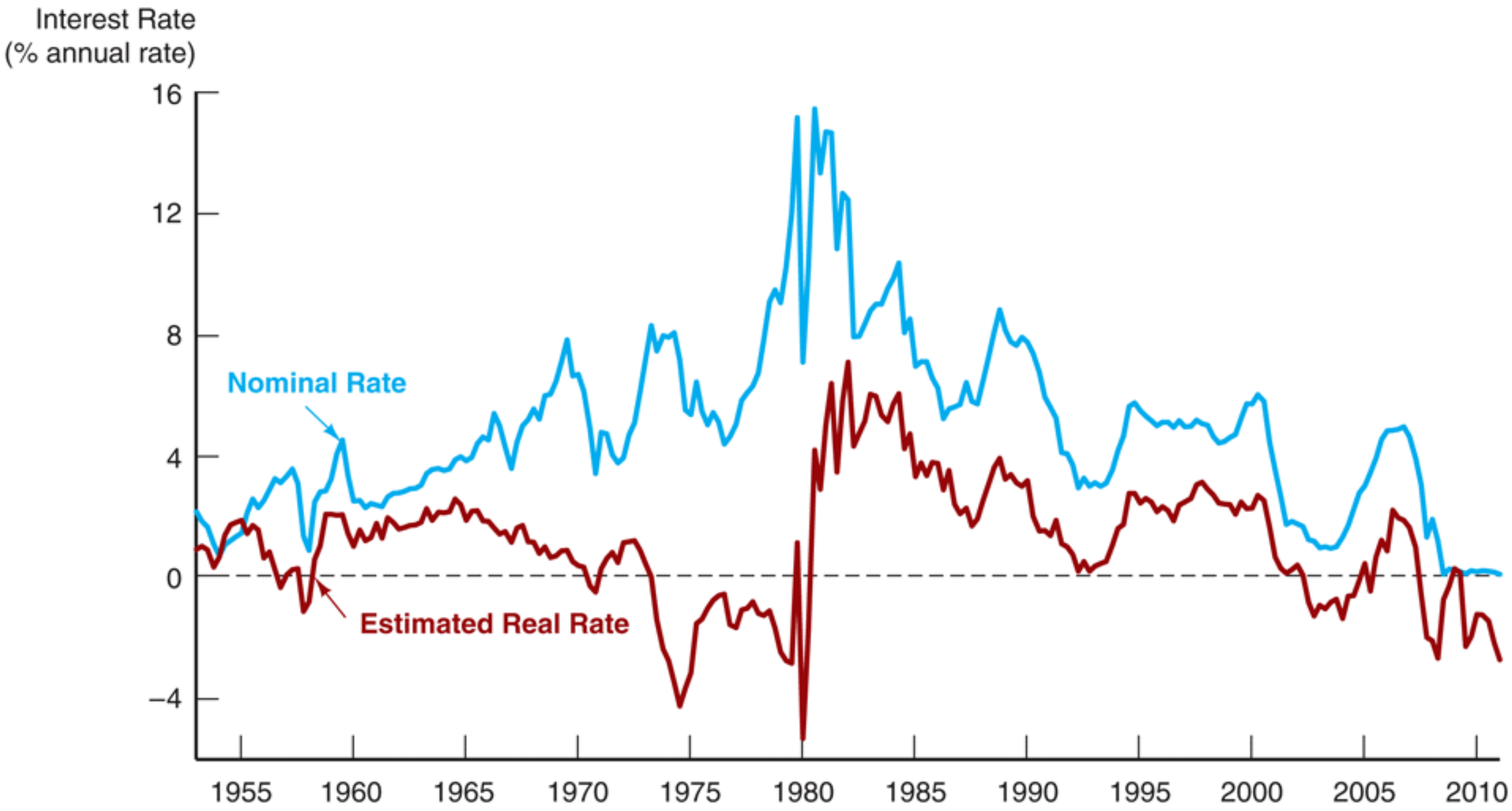
π^e = expected inflation rate

When the real interest rate is low,

there are greater incentives to borrow and fewer incentives to lend.

The real interest rate is a better indicator of the incentives to borrow and lend.

Figure 1 Real and Nominal Interest Rates (Three-Month Treasury Bill), 1953–2011



Sources: Nominal rates from www.federalreserve.gov/releases/H15 and inflation from ftp://ftp.bis.gov/special.requests/cpi/cpia.txt. The real rate is constructed using the procedure outlined in Frederic S. Mishkin, "The Real Interest Rate: An Empirical Investigation," *Carnegie-Rochester Conference Series on Public Policy 15 (1981): 151–200. This procedure involves estimating* expected inflation as a function of past interest rates, inflation, and time trends and then subtracting the expected inflation measure from the nominal interest rate.