SISTEM **DIGITAL**

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BAB 4:
PENYEDERHANAAN **PERSAMAAN** LOGIKA

PENYEDERHANAAN PERSAMAAN LOGIKA

Capaian Pembelajaran

- Mahasiswa mampu menyederhanakan persamaan logika menggunakan ***Karnaugh Map (K-Map)***.
- Mahasiswa mampu menyederhanakan rangkaian digital pada gate level dengan metode tabulasi.

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PENYEDERHANAAN PERSAMAAN LOGIKA

- Kerumitan rangkaian gerbang logika bergantung pada fungsi Boolean
- **Mengapa perlu disederhanakan?**
  - rangkaian logika yang lebih sederhana
  - biaya lebih rendah, asupan tegangan lebih kecil, dan cepat.

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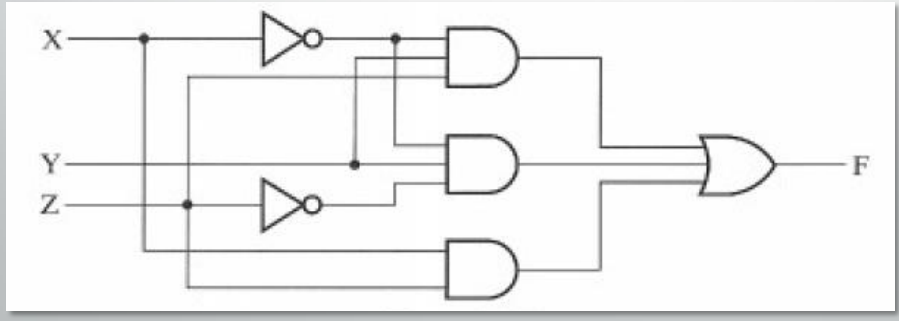
PENYEDERHANAAN PERSAMAAN LOGIKA

Contoh.
Buatlah rangkaian gerbang logika untuk persaamaan boolean **F = X'YZ + X'YZ' + XZ**



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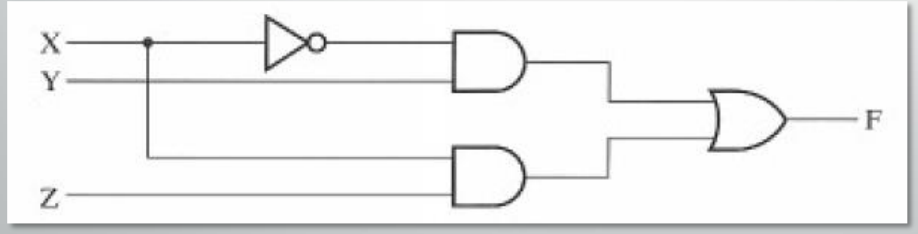
PENYEDERHANAAN PERSAMAAN LOGIKA

Disederhanakan menjadi:

F = X'YZ + X'YZ' + XZ
= X'Y(Z + Z') + XZ
= X'Y + XZ



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KARNAUGH MAPS (K-MAPS)

- Sebuah metode sistematis untuk menyederhanakan notasi SOP (sum-of-product)
- Tujuan penyederhanaan → meminimalisasi literal
- Direpresentasikan secar grafis
- Kelebihan: lebih mudah digunakan
- Kekurangan: terbatas hanya 5 sampai 6 variabel

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PENYEDERHANAAN PERSAMAAN LOGIKA

- **Teknik penyederhanaannya?**
  - Algebraic
    - Using theorems
    - Open-ended; requires skills
  - Karnaugh Maps
    - Easy to use
    - Limited to no more than 6 variables
  - Quine-McCluskey
    - Suitable for automation
    - Can handle many variables (but computationally intensive)

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PENYEDERHANAAN PERSAMAAN LOGIKA

Contoh.
Sederhanakan rangkaian gerbang logika untuk persamaan boolean **F = y'z' + xy + x'yz**

Jawab.

F = y'z' + xy + x'yz
= y'z' + xy.(1 + z) + x'yz
= y'z' + xy + xyz + x'yz
= y'z' + xy + yz . (x + x')
= y'z' + xy + yz

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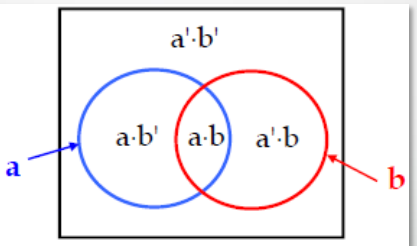
KARNAUGH MAPS (K-MAPS)

- Example: 2 variables a and b represented by 2 circular regions. There are 4 minterms, each occupying their respective space.



{ a·b, a·b' } →
{ a'·b, a·b } →
{ a·b } →
{ a·b, a·b', a'·b } →
{ } →
{ a'·b', a·b, a·b', a'·b } →

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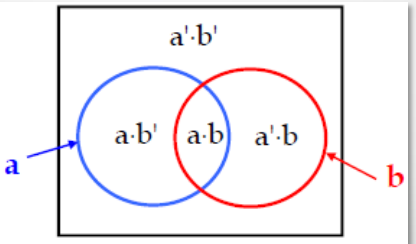
KARNAUGH MAPS (K-MAPS)

- Example: 2 variables a and b represented by 2 circular regions. There are 4 minterms, each occupying their respective space.



{ a·b, a·b' } →
{ a'·b, a·b } →
{ a·b } →
{ a·b, a·b', a'·b } →
{ } →
{ a'·b', a·b, a·b', a'·b } →

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KARNAUGH MAPS (K-MAPS)

K-Maps 2 Variabel.

	B'	B
A'	A'B'	A'B
A	AB'	AB

⇨

	B'	B
A'	00	01
A	10	11

⇨

	B'	B
A'	m0	m1
A	m2	m3

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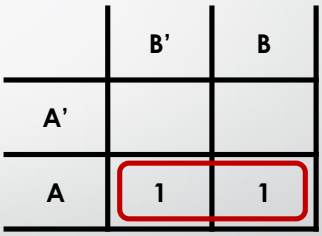
KARNAUGH MAPS (K-MAPS)

Contoh.

F = A . B

F = **?**

F = A'B'



F = **?**

F = A

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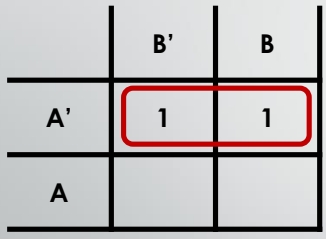
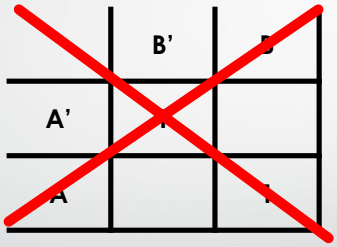
KARNAUGH MAPS (K-MAPS)

Contoh.



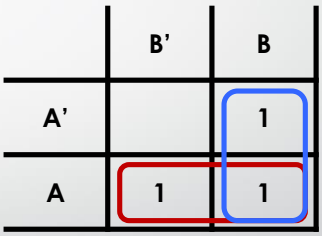
F = **?**

F = A'



F = **?**

Latihan.



F = **?**

F = A + B

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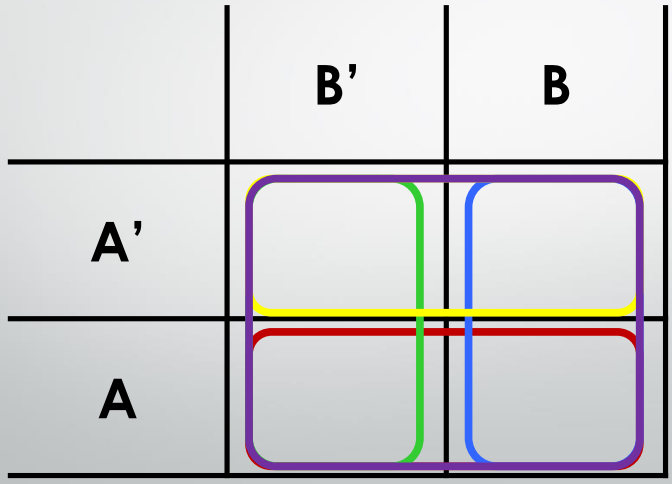
KARNAUGH MAPS (K-MAPS)



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KARNAUGH MAPS (K-MAPS)

K-Maps 3 Variabel.

	B'C'	B'C	BC	BC'
A'	A'B'C'	A'B'C	A'BC	A'BC'
A	AB'C'	AB'C	ABC	ABC'

⇨

	B'C'	B'C	BC	BC'
A'	m0	m1	m3	m2
A	m4	m5	m7	m6

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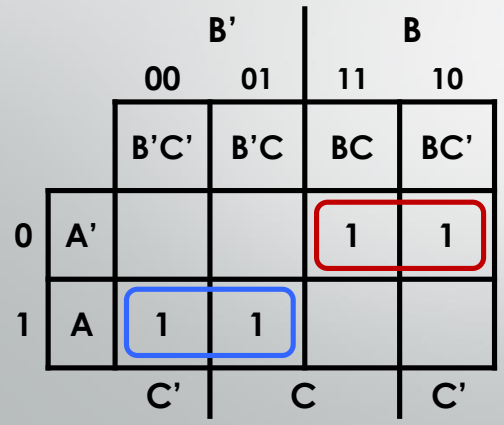
KARNAUGH MAPS (K-MAPS)

Contoh.



F = **?**

A'BC + A'BC'

A'B . (C + C')

A'B . 1 → A'B

AB'C' + AB'C

AB' . (C + C')

AB' . 1 → AB'

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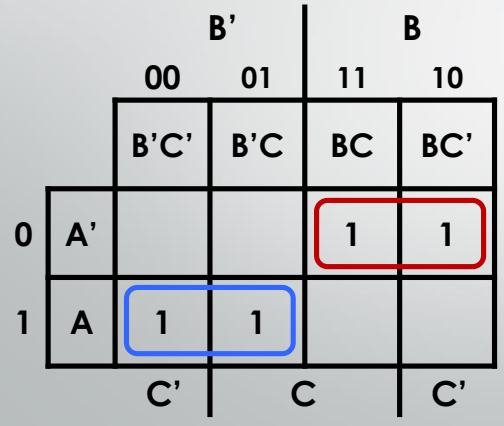
KARNAUGH MAPS (K-MAPS)

Contoh.



F = **A'B + AB'**

A'BC + A'BC'

A'B . (C + C')

A'B . 1 → A'B

AB'C' + AB'C

AB' . (C + C')

AB' . 1 → AB'

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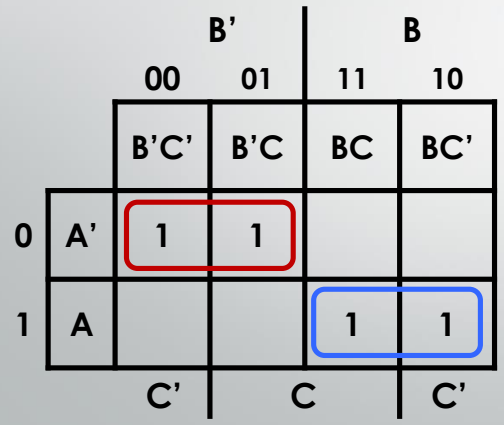
KARNAUGH MAPS (K-MAPS)

Contoh.



F = ?

A'B'C' + A'B'C

A'B' . (C + C')

A'B' . 1 → A'B'

ABC + ABC'

AB . (C + C')

AB . 1 → AB

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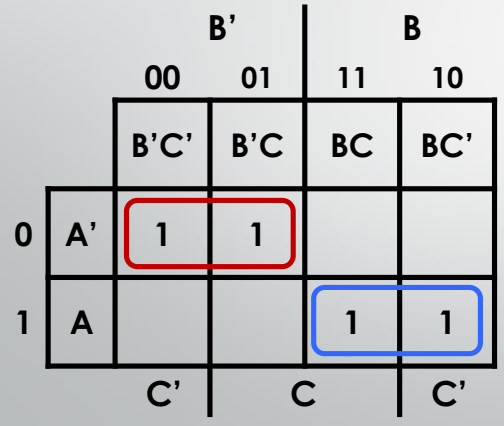
KARNAUGH MAPS (K-MAPS)

Contoh.



F = **A'B' + AB**

A'B'C' + A'B'C

A'B' . (C + C')

A'B' . 1 → A'B'

ABC + ABC'

AB . (C + C')

AB . 1 → AB

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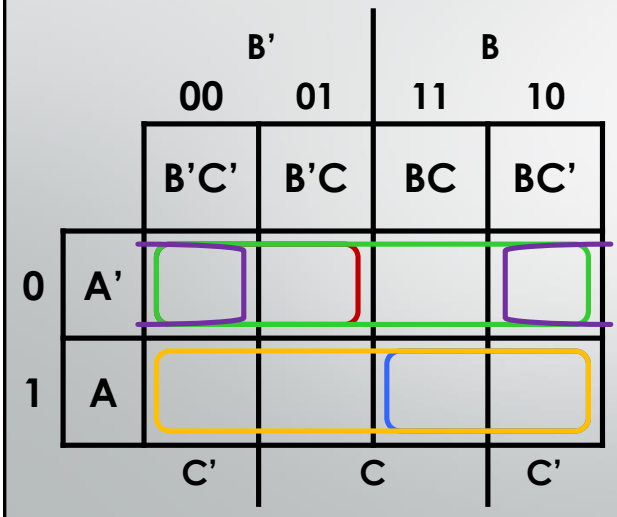
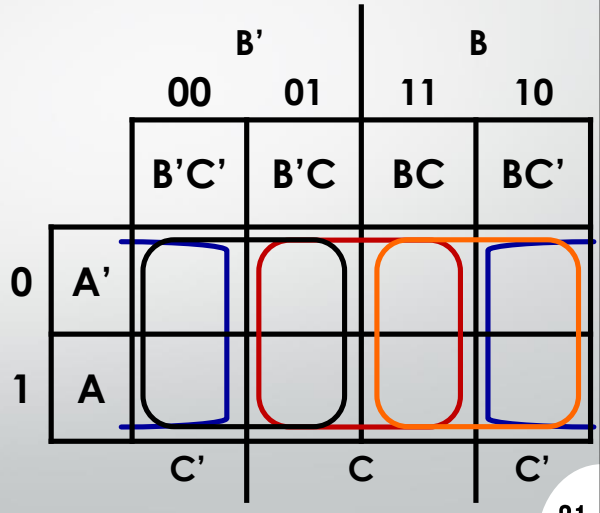
KARNAUGH MAPS (K-MAPS)





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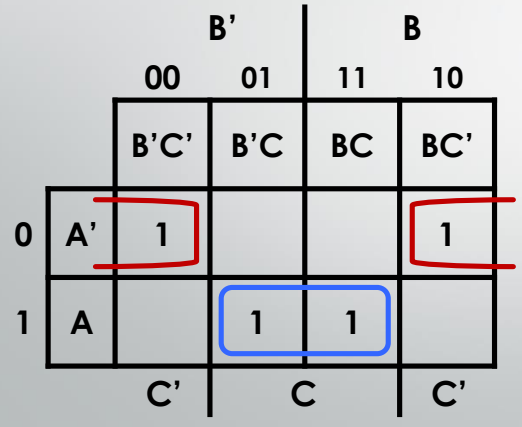
KARNAUGH MAPS (K-MAPS)

Contoh.



F = **?**

A'B'C' + A'BC'

A'C' . (B + B')

A'C' . 1 → A'C'

AB'C + ABC

AC . (B + B')

AC . 1 → AC

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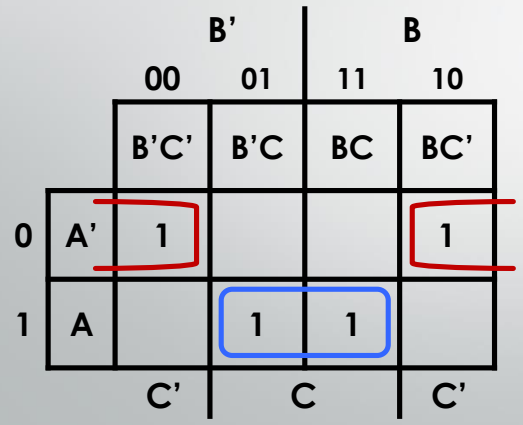
KARNAUGH MAPS (K-MAPS)

Contoh.



F = **A'C' + AC**

A'B'C' + A'BC'

A'C' . (B + B')

A'C' . 1 → A'C'

AB'C + ABC

AC . (B + B')

AC . 1 → AC

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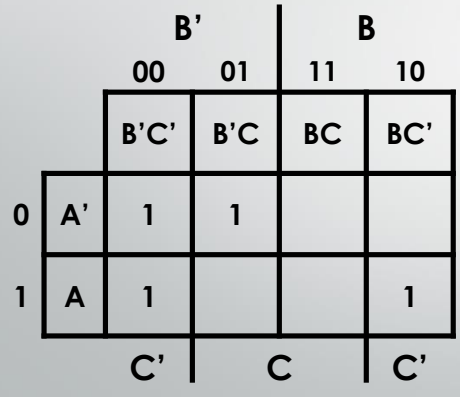
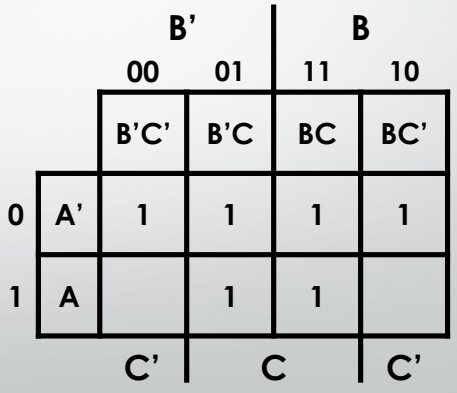
KARNAUGH MAPS (K-MAPS)

Latihan #1.



Latihan #2.



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KARNAUGH MAPS (K-MAPS)

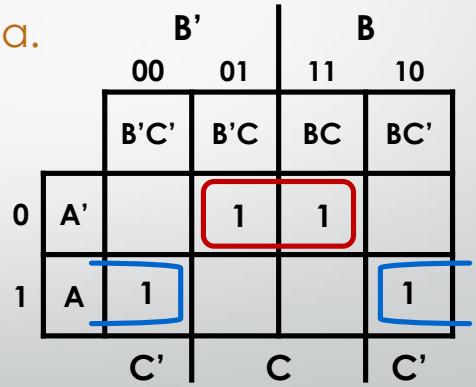
Latihan #3.

a. Tentukan K-Maps untuk **minterm** F(xyz) = $\sum_m(1,3,4,6)$

b. Tuliskan persamaan ***boolean*** yang terbentuk dari k-maps tersebut

Jawab.

a.



b. **F = A'C + AC'**

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KARNAUGH MAPS (K-MAPS)

Latihan #4.

Sederhanakan fungsi Boolean berikut menggunakan K-Maps.

1. F = X'YZ + X'YZ' + XY'Z' + XY'Z
2. F = X'YZ + XY'Z' + XYZ + XYZ'
3. F = A'C + A'B + AB'C + BC
4. F = ∑m (2,3,4,5,7)
5. F = ∑m (0,1,2,3,4,5,7)

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KARNAUGH MAPS (K-MAPS)

Jawab.

1. F = X'Y + XY'
2. F = YZ + XZ'
3. F = C + A'B
4. F = X'Y + XY + XZ
5. F = X' + Y' + Z

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KARNAUGH MAPS (K-MAPS)

K-Maps 4 Variabel.

	C'D'	C'D	CD	CD'
A'B'	A'B'C'D'	A'B'C'D	A'B'CD	A'B'CD'
A'B	A'BC'D'	A'BC'D	A'BCD	A'BCD'
AB	ABC'D'	ABC'D	ABCD	ABCD'
AB'	AB'C'D'	AB'C'D	AB'CD	AB'CD'

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KARNAUGH MAPS (K-MAPS)

K-Maps 4 Variabel.

	C'D'	C'D	CD	CD'
A'B'	0000	0001	0011	0010
A'B	0100	0101	0111	0110
AB	1100	1101	1111	1110
AB'	1000	1001	1011	1010

	C'D'	C'D	CD	CD'
A'B'	m0	m1	m3	m2
A'B	m4	m5	m7	m6
AB	m12	m13	m15	m14
AB'	m8	m9	m11	m10

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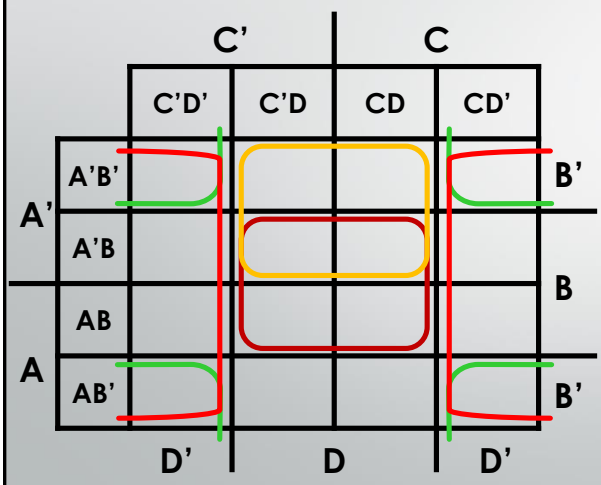
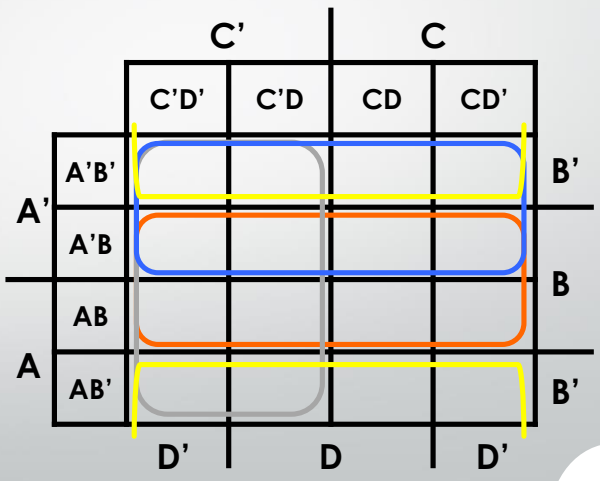
KARNAUGH MAPS (K-MAPS)





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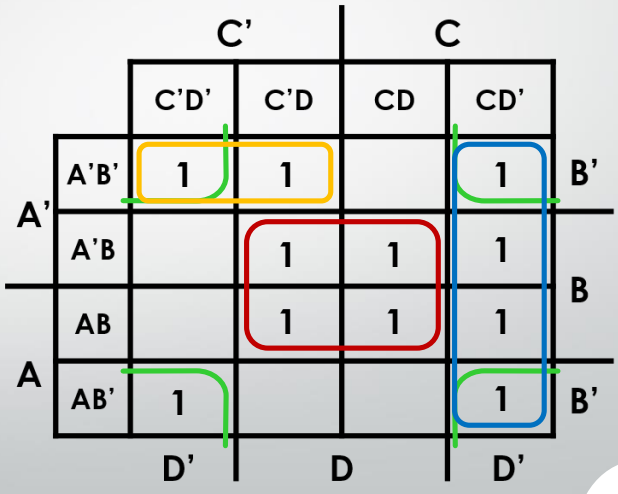
KARNAUGH MAPS (K-MAPS)

Contoh.

Tuliskan persamaan ***boolean*** k-maps berikut!

Jawab.

F = B'D' + A'B'C' + BD + CD'



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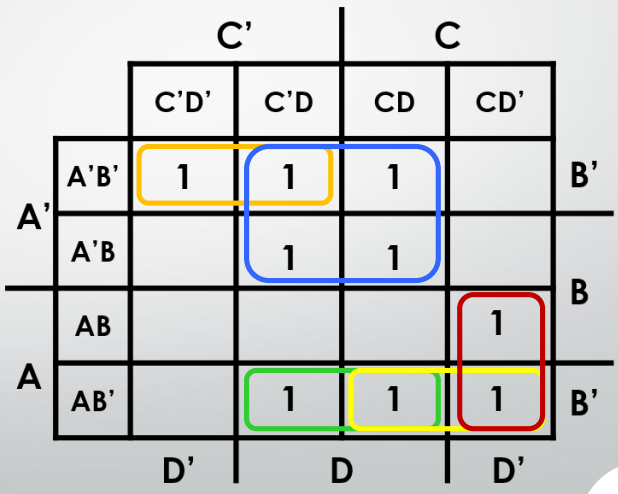
KARNAUGH MAPS (K-MAPS)

Latihan #1.

Tuliskan persamaan ***boolean*** k-maps berikut!

Jawab.

F = A'B'C' + AD + AB'D + A'BC' + ACD'



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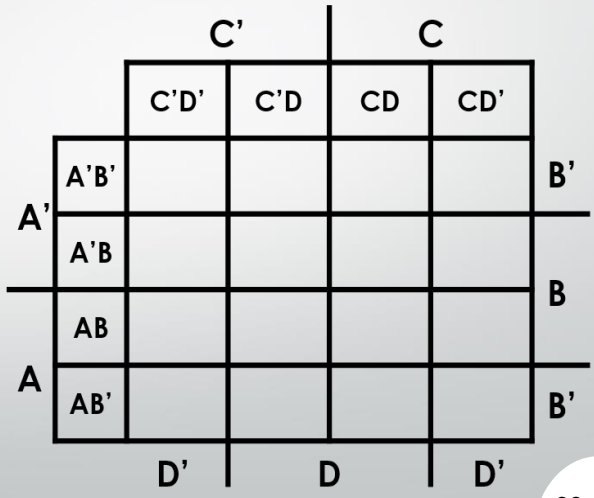
KARNAUGH MAPS (K-MAPS)

Latihan #2.

a. Tentukan k-maps untuk **minterm** F(wxyz) = $\sum_m(0,2,4,5,6,7,9,11,12,13)$
b. Tuliskan persamaan ***boolean*** yang terbentuk dari k-maps tersebut.



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KARNAUGH MAPS (K-MAPS)

Latihan #3.

Sederhanakan fungsi Boolean berikut menggunakan K-Maps.

1. F(w,x,y,z)=∑m(0,1,5,7,13,15)
2. F(w,x,y,z)=∑m(0,1,2,4,5,6,8,9,12,13,14)
3. F = A'B'C'+B'CD'+A'BCD'+AB'C'

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KARNAUGH MAPS (K-MAPS)

Jawab.

1. F = Z + W'X'Y'
2. F = Y' + W'Z' + XZ'
3. F = A'CD' + B'C' + A'B'D'

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KARNAUGH MAPS (K-MAPS)

K-Maps 5 Variabel.

A'

	D'E'	D'E	DE	DE'
B'C'	m0	m1	m3	m2
B'C	m4	m5	m7	m6
BC	m12	m13	m15	m14
BC'	m8	m9	m11	m10

A

	D'E'	D'E	DE	DE'
B'C'	m16	m17	m19	m18
B'C	m20	m21	m23	m22
BC	m24	m25	m27	m26
BC'	m28	m29	m31	m30

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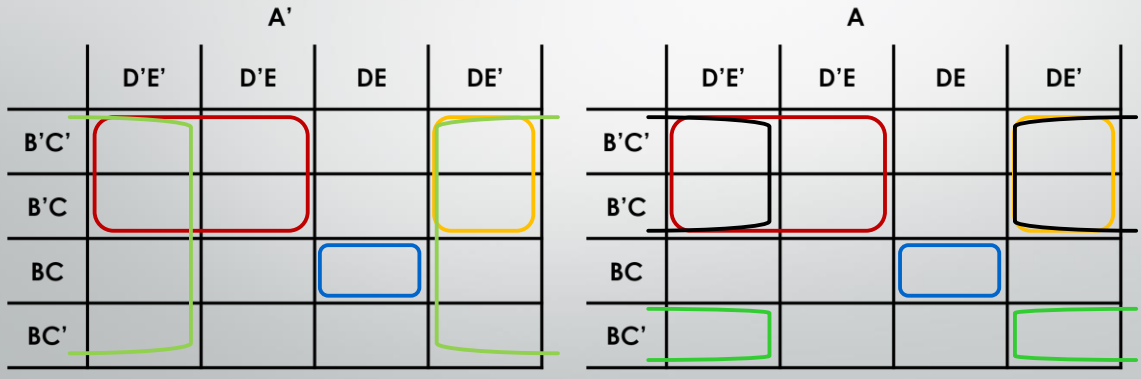
KARNAUGH MAPS (K-MAPS)

K-Maps 5 Variabel.



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KARNAUGH MAPS (K-MAPS)

Contoh.

Sederhanakan fungsi ***boolean*** berikut dengan K-Maps

1. F(A,B,C,D,E) = ∑m(0,2,4,7,10,12,13,18,23,26,28,29)
2. F(A,B,C,D,E)= ∑m(0,1,2,5,6,7,8,10,13,14,15,16,17,21,23,24,25,28,29, 30,31)

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KARNAUGH MAPS (K-MAPS)

3. F(A,B,C,D,E)= ∑m(0,2,4,8,10,12,13,14,15,16,18,20,21,23,24,26,29,30, 31)

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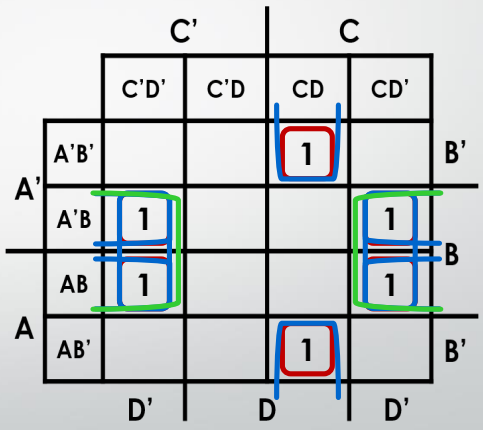
IMPLICANT

- **Implicant:** Semua kelompok minterm yang memenuhi persamaan 2^n
  - 6 implicant 1 minterm (**merah**)
  - 5 implicant 2 minterm (**biru**)
  - 1 implicant 4 minterm (**hijau**)
  - Total = 12 implicant



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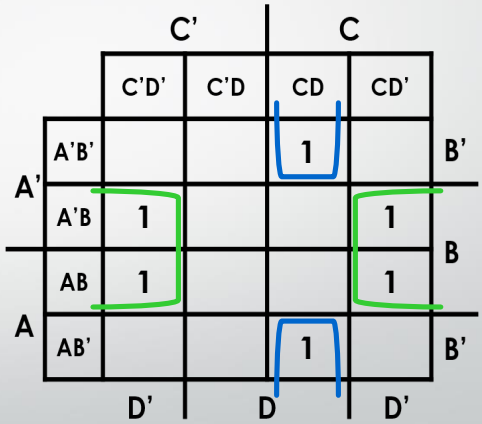
PRIME IMPLICANTS (PIs)

- **Prime implicant (PI):** Kelompok implicant terbesar yang dapat digambarkan pada K-Maps
- Sebuah implicant dikatakan ***prime*** apabila tidak ada implicant lain yang menutupinya



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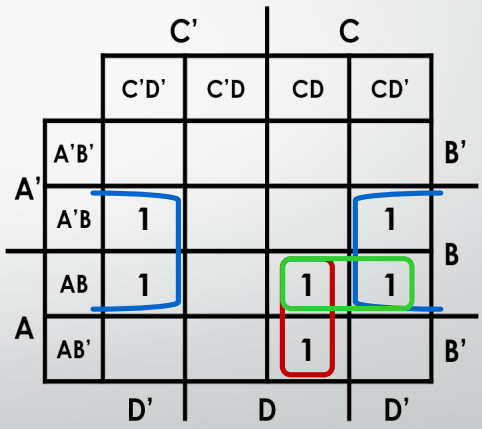
ESSENTIAL PRIME IMPLICANTS (EPIs)

- Essential minterm adalah minterm yang ditutupi hanya oleh 1 PI
- Minterm tersebut dinamakan **Essential PI** (**EPI**)
- Gambar disamping memperlihatkan bahwa warna **merah** dan **biru** merupakan EPI
- Warna **hijau** bukan EPI karena salah satu atau kedua ***product term***-nya telah menjadi bagian EPI



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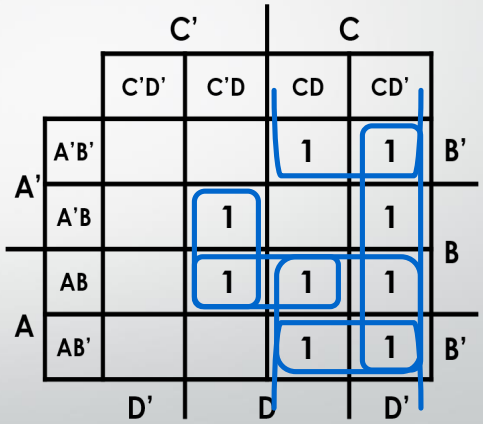
ESSENTIAL PRIME IMPLICANTS (EPIs)

Contoh.

Jika semua PI diperlihatkan seperti gambar di samping, maka tentukan:

1. semua PI yang teridentifikasi;
2. semua EPI yang teridentifikasi;
3. K-Maps hasil penyederhanaan.



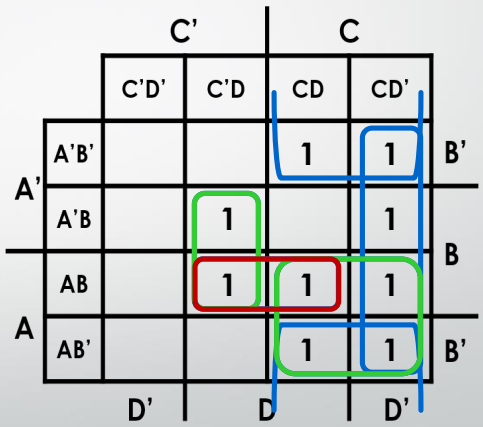
43

ESSENTIAL PRIME IMPLICANTS (EPIs)

Contoh.

Jika semua PI diperlihatkan seperti gambar di samping, maka tentukan:

1. semua PI yang teridentifikasi;
2. semua EPI yang teridentifikasi;
3. K-Maps hasil penyederhanaan.



44

ESSENTIAL PRIME IMPLICANTS (EPIs)

Contoh.

Jika semua PI diperlihatkan seperti gambar di samping, maka tentukan:

1. semua PI yang teridentifikasi;
2. semua EPI yang teridentifikasi;
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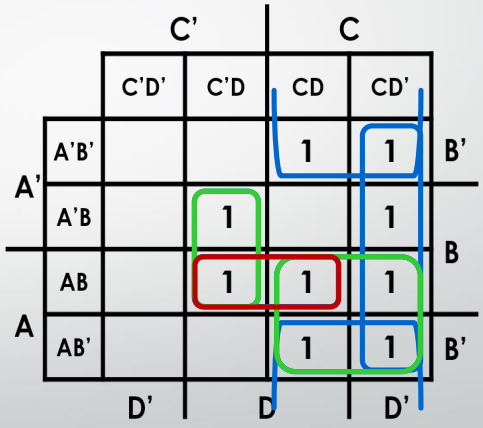

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IMPLICANTs, PIs, & EPIs

Tugas.

Temukan semua ***prime implicant*** pada fungsi boolean berikut dan sederhanakanlah:

1. F(A,B,C,D) = ∑m (0,1,2,4,5,7,10,12,13,15)
2. F(W,X,Y,Z) = ∑m (1,3,4,5,6,7,8,9,10,11,12,14)
3. F(A,B,C,D,E) = ∑m (2,3,4,5,6,10,11,12,13,14,16,20,21,24,25,26,27,28,29)

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KONDISI DON'T CARE

- Kondisi **don't care:** Suatu kondisi yang dapat diasumsikan mempunyai keadaan 0 atau 1 yang juga ditandai dengan X dan untuk menyederhanakan ekspresi boolean menggunakan K-Maps.

  Contoh.
  Sederhanakan fungsi Boolean berikut:
  F(A,B,C,D) = ∑m (1,3,7,11,15), yang mempunyai kondisi **don't care**: d(A,B,C,D) = ∑m (0,2,5)

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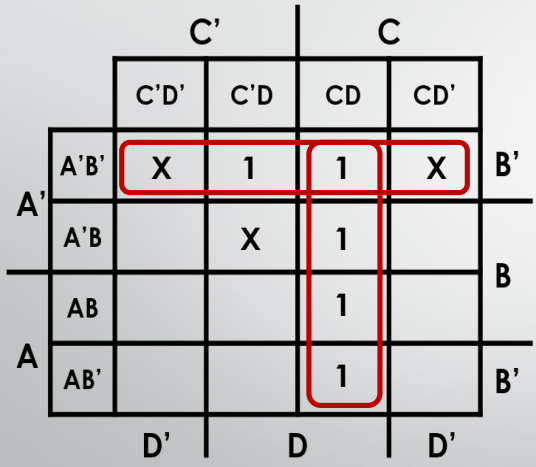
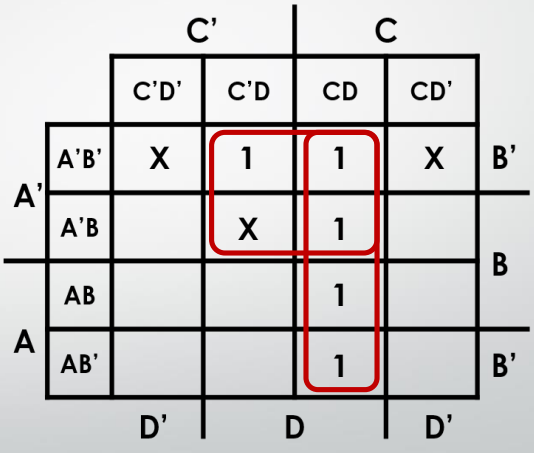
KONDISI DON'T CARE



F = A'B' + CD



F = A'D + CD

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KONDISI DON'T CARE

Contoh.

Sederhanakan fungsi Boolean berikut:
F(A,B,C,D) = ∑m (4,5,6,7,13) yang mempunyai kondisi **don't care**: d(A,B,C,D) = ∑m (1,9,15).

Soal di atas biasanya ditulis dengan:
F(A,B,C,D) = ∑m (4,5,6,7,13) + d (1,9,15), dimana d adalah minterm lokasi ***don't care***

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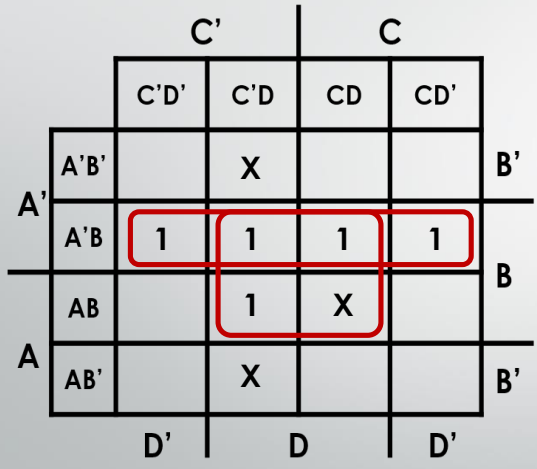
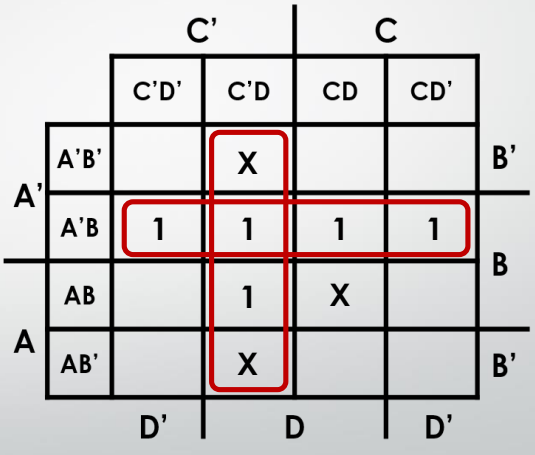
KONDISI DON'T CARE



F = A'B + BD



F = A'B + C'D

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KONDISI DON'T CARE

Tugas.

Temukan semua ***prime implicant*** pada fungsi boolean + kondisi ***don't care*** berikut dan sederhanakanlah:

1. F(A,B,C,D) = ∑m (3,4,7,13,14) + d (5,9,15)
2. F(A,B,C,D,E) = ∑m (2,3,4,5,10,12,14,18,19,21,27,28, 30) + d(8,13,20,26,29)

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QUINE-McCLUSKEY METHOD

- **Quine-McCluscky method** pada umunya dikenal dengan nama metode penyederhanaan persamaan boolean metode tabulasi.
- ***Mengapa digunakan (juga) metode ini?***
  - Karnaugh Maps (K-Map), sangat efektif untuk meminimumkan persamaan aljabar boolean untuk 1 s.d. 4 input.
  - Untuk input yang lebih besar dari 4 sudah mulai sulit dengan menggunakan K-Map.

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QUINE-McCLUSKEY METHOD

- K-Map sangat tergantung dari kemampuan seseorang untuk melihat ***prime implicant* (PI)**dan memilih sejumlah ***prime implicant*** yang dapat mencakup ***minterm*** sebanyak-banyaknya.

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QUINE-McCLUSKEY METHOD

Willard van Orman Quine 1908-2000, Edgar Pierce Chair of Philosophy at Harvard University.

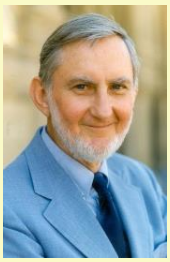
http://members.aol.com/drquine/wv-quine.html

Quine, Willard, "The problem of simplifying truth functions." *American Mathematical Monthly, vol. 59, 1952.*

Quine, Willard, "A way to simplify truth functions." *American Mathematical Monthly, vol. 62, 1955.*

Edward J. McCluskey, Professor of Electrical Engineering and Computer Science at Stanford

http://www-crc.stanford.edu/users/ejm/McCluskey_Edward.html



McCluskey Jr., Edward J. "Minimization of Boolean Functions." *Bell Systems Technical Journal, vol. 35, pp. 1417-1444, 1956*

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QUINE-McCLUSKEY METHOD

Tahapan metode Quine-McCluskey.

- Buat persamaan boolean ke dalam bentuk standard sum-of-minterm
- Eliminasi sebanyak mungkin literal, dengan menerapkan hukum boolean: **xy+xy'= x**.
- Gambarkan ***prime inplicant chart*** untuk memilih ***minimum set of prime implicant***

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QUINE-McCLUSKEY METHOD

Menetukan PIs.

- AB'CD' + AB'CD = AB'C _ atau AB'C -
  1 0 1 0 + 1 0 1 1 = 1 0 1 _ atau 1 0 1 -
- Tanda " _ " atau " - ": adalah variabel yang dihilangkan
- Ke-2 minterm diatas dapat digabungkan karena memiliki perbedaan satu bit
- A'BC'D + A'BCD', tidak dapat digabungkan karena memiliki perbedaan > 1 bit

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QUINE-McCLUSKEY METHOD

Contoh.
Temukan semua ***prime implicant*** dari:
F(A,B,C,D) = ∑m (0,1,2,5,6,7,8,9,10,14)

Tugas.
Temukan semua ***prime implicant*** dari:
F(A,B,C,D,E) =
∑m (0,2,4,5,6,7,8,9,10,11,13,15,21,23,26,28,29,30,31)

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BAB 4:
PENYEDERHANAAN **PERSAMAAN** LOGIKA

Terima Kasih